

Surds

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the expression equal to $3\sqrt{5}$

Options: $\sqrt{8}$, $\sqrt{15}$, $\sqrt{45}$, $\sqrt{75}$

Write the 3 as a square root: $3 = \sqrt{9}$

$$3\sqrt{5} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$$

Answer: $\sqrt{45}$

Tip: To move a number inside a square root, square it first: $3\sqrt{5} = \sqrt{3^2 \times 5}$, not $\sqrt{15}$.

2 (2 marks)

Simplify fully $2\sqrt{63} - \sqrt{28}$

Both numbers are 7 times a square number.

$$\sqrt{63} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}, \text{ so } 2\sqrt{63} = 6\sqrt{7}$$

$$\sqrt{28} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$$

$$6\sqrt{7} - 2\sqrt{7} = 4\sqrt{7}$$

Answer: $4\sqrt{7}$

Tip: Simplify $\sqrt{63}$ first, then multiply by the 2 in front: $2 \times 3\sqrt{7} = 6\sqrt{7}$. You can only subtract surds once they are multiples of the same root.

Marks: 1 mark for $3\sqrt{7}$ (or $6\sqrt{7}$) or $2\sqrt{7}$. 1 mark for $4\sqrt{7}$.

3 (2 marks)

Write $\frac{18}{\sqrt{6}} + \sqrt{54}$ in the form $a\sqrt{6}$, where a is an integer.

Rationalise the fraction: multiply top and bottom by $\sqrt{6}$

$$\frac{18}{\sqrt{6}} = \frac{18\sqrt{6}}{6} = 3\sqrt{6}$$

$$\sqrt{54} = \sqrt{9} \times \sqrt{6} = 3\sqrt{6}$$

$$3\sqrt{6} + 3\sqrt{6} = 6\sqrt{6}$$

Answer: $6\sqrt{6}$

Tip: $\sqrt{6} \times \sqrt{6} = 6$, not 36. So $\frac{18}{\sqrt{6}} = \frac{18\sqrt{6}}{6}$.

Marks: 1 mark for $3\sqrt{6}$ from either term. 1 mark for $6\sqrt{6}$.

4 (4 marks)

Expand and simplify

(a) $(5 - 2\sqrt{3})(1 + \sqrt{3})$

Multiply each term in the first bracket by each term in the second:

$$5 \times 1 = 5 \quad 5 \times \sqrt{3} = 5\sqrt{3}$$

$$-2\sqrt{3} \times 1 = -2\sqrt{3} \quad -2\sqrt{3} \times \sqrt{3} = -2 \times 3 = -6$$

$$\text{Collect like terms: } 5 - 6 + 5\sqrt{3} - 2\sqrt{3} = -1 + 3\sqrt{3}$$

Answer: $-1 + 3\sqrt{3}$

Tip: $-2\sqrt{3} \times \sqrt{3} = -6$. The surds multiply to give 3, then multiply by -2 .

Marks: 1 mark for four correct terms. 1 mark for $-1 + 3\sqrt{3}$.

(b) $(\sqrt{7} - \sqrt{2})^2$

Write it as two brackets: $(\sqrt{7} - \sqrt{2})(\sqrt{7} - \sqrt{2})$

$$\sqrt{7} \times \sqrt{7} = 7 \quad \sqrt{2} \times \sqrt{2} = 2$$

The two middle terms: $-\sqrt{7} \times \sqrt{2} = -\sqrt{14}$, twice, so $-2\sqrt{14}$

Total: $7 + 2 - 2\sqrt{14} = 9 - 2\sqrt{14}$

Answer: $9 - 2\sqrt{14}$

Tip: $(a - b)^2$ has a middle term: $(\sqrt{7} - \sqrt{2})^2$ is not $7 - 2 = 5$.

Marks: 1 mark for 7, 2 and $-\sqrt{14}$ (twice) seen. 1 mark for $9 - 2\sqrt{14}$.

5 (2 marks)

Rationalise the denominator and simplify fully $\frac{6}{4 - \sqrt{10}}$

Multiply top and bottom by the conjugate $4 + \sqrt{10}$ (change the sign).

Denominator: $(4 - \sqrt{10})(4 + \sqrt{10}) = 16 - 10 = 6$

Numerator: $6(4 + \sqrt{10})$

$$\frac{6(4 + \sqrt{10})}{6} = 4 + \sqrt{10}$$

Answer: $4 + \sqrt{10}$

Tip: $(4 - \sqrt{10})(4 + \sqrt{10}) = 16 - 10$, not $16 + 10$. The middle terms cancel, and $\sqrt{10} \times \sqrt{10}$ is subtracted.

Marks: 1 mark for multiplying top and bottom by $4 + \sqrt{10}$. 1 mark for $4 + \sqrt{10}$.

6 (3 marks)

Simplify fully $\frac{\sqrt{112} - \sqrt{28}}{\sqrt{63}}$

Each number is 7 times a square number.

$$\sqrt{112} = \sqrt{16} \times \sqrt{7} = 4\sqrt{7}$$

$$\sqrt{28} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$$

$$\sqrt{63} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$$

So the fraction is $\frac{4\sqrt{7} - 2\sqrt{7}}{3\sqrt{7}} = \frac{2\sqrt{7}}{3\sqrt{7}}$

Cancel the $\sqrt{7}$: $\frac{2}{3}$

Answer: $\frac{2}{3}$

Tip: You cannot do $\sqrt{112} - \sqrt{28} = \sqrt{84}$. Simplify each surd to a multiple of $\sqrt{7}$ first, then subtract.

Marks: 1 mark for two of $4\sqrt{7}$, $2\sqrt{7}$, $3\sqrt{7}$. 1 mark for $\frac{2\sqrt{7}}{3\sqrt{7}}$. 1 mark for $\frac{2}{3}$.

7 (3 marks)

Solve $x^2 + 4x - 14 = 0$

Give your answers in the form $a \pm b\sqrt{c}$, where a , b and c are integers.

Complete the square: $x^2 + 4x = (x + 2)^2 - 4$

So $(x + 2)^2 - 4 - 14 = 0$, which gives $(x + 2)^2 = 18$

Square root both sides (remember $\pm$): $x + 2 = \pm\sqrt{18}$

Simplify: $\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$x = -2 \pm 3\sqrt{2}$

(The quadratic formula gives $x = \frac{-4 \pm \sqrt{72}}{2} = \frac{-4 \pm 6\sqrt{2}}{2}$, the same answer.)

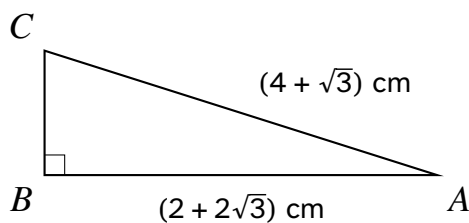
Answer: $x = -2 \pm 3\sqrt{2}$

Tip: Simplify the surd fully: $-2 \pm \sqrt{18}$ is not finished, because 18 has the square factor 9. And don't forget the $\pm$: there are two answers.

Marks: 1 mark for $(x + 2)^2 = 18$ or a correct substitution into the formula. 1 mark for $\sqrt{18} = 3\sqrt{2}$ (or $\sqrt{72} = 6\sqrt{2}$). 1 mark for $-2 \pm 3\sqrt{2}$.

8 (5 marks)

ABC is a right-angled triangle.



Not drawn accurately

(a) Show that $BC = \sqrt{3}$ cm.

Pythagoras: $BC^2 = AC^2 - AB^2$

$$AC^2 = (4 + \sqrt{3})^2 = 16 + 8\sqrt{3} + 3 = 19 + 8\sqrt{3}$$

$$AB^2 = (2 + 2\sqrt{3})^2 = 4 + 8\sqrt{3} + 12 = 16 + 8\sqrt{3}$$

$$BC^2 = (19 + 8\sqrt{3}) - (16 + 8\sqrt{3}) = 3$$

$$BC = \sqrt{3} \text{ cm } \checkmark$$

Answer: $BC^2 = 3$, so $BC = \sqrt{3}$ cm

Tip: Bracket the whole of AB^2 before subtracting it, so that both terms are taken away.
And $(2\sqrt{3})^2 = 4 \times 3 = 12$.

Marks: 1 mark for $AC^2 - AB^2$ with brackets. 1 mark for $19 + 8\sqrt{3}$ or $16 + 8\sqrt{3}$. 1 mark for $BC^2 = 3$ leading to $\sqrt{3}$.

(b) Work out the area of triangle ABC .

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (2 + 2\sqrt{3}) \times \sqrt{3}$$

$$(2 + 2\sqrt{3}) \times \sqrt{3} = 2\sqrt{3} + 6$$

Half of this: $3 + \sqrt{3}$

Answer: $3 + \sqrt{3} \text{ cm}^2$

Tip: Use the two sides at the right angle (AB and BC) as the base and height, not the hypotenuse.

Marks: 1 mark for $\frac{1}{2} \times (2 + 2\sqrt{3}) \times \sqrt{3}$. 1 mark for $3 + \sqrt{3}$.

9 (4 marks)

Rationalise the denominator and simplify fully $\frac{9 + 4\sqrt{3}}{2\sqrt{3} - 3}$
Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

Multiply top and bottom by the conjugate $2\sqrt{3} + 3$

Denominator: $(2\sqrt{3} - 3)(2\sqrt{3} + 3) = 12 - 9 = 3$

Numerator: $(9 + 4\sqrt{3})(2\sqrt{3} + 3)$

$= 18\sqrt{3} + 27 + 8 \times 3 + 12\sqrt{3}$

$= 18\sqrt{3} + 27 + 24 + 12\sqrt{3} = 51 + 30\sqrt{3}$

Divide by 3: $\frac{51 + 30\sqrt{3}}{3} = 17 + 10\sqrt{3}$

Answer: $17 + 10\sqrt{3}$

Tip: $(2\sqrt{3})^2 = 4 \times 3 = 12$, so the denominator is $12 - 9 = 3$. Divide **both** terms of the numerator by 3 at the end.

Marks: 1 mark for multiplying by $2\sqrt{3} + 3$. 1 mark for the denominator 3. 1 mark for $51 + 30\sqrt{3}$. 1 mark for $17 + 10\sqrt{3}$.

10 (4 marks)

Solve $x\sqrt{5} + 3 = 2x + \sqrt{5}$

Give your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Collect the x terms on one side: $x\sqrt{5} - 2x = \sqrt{5} - 3$

Factorise: $x(\sqrt{5} - 2) = \sqrt{5} - 3$

$$x = \frac{\sqrt{5} - 3}{\sqrt{5} - 2}$$

Multiply top and bottom by $\sqrt{5} + 2$

$$\text{Denominator: } (\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$$

$$\text{Numerator: } (\sqrt{5} - 3)(\sqrt{5} + 2) = 5 + 2\sqrt{5} - 3\sqrt{5} - 6 = -1 - \sqrt{5}$$

$$x = -1 - \sqrt{5}$$

Answer: $x = -1 - \sqrt{5}$

Tip: Treat $\sqrt{5}$ like any other number: get the x terms together and factorise out x . Then rationalise the denominator to reach the form asked for.

Marks: 1 mark for $x(\sqrt{5} - 2) = \sqrt{5} - 3$. 1 mark for $x = \frac{\sqrt{5} - 3}{\sqrt{5} - 2}$. 1 mark for multiplying by $\sqrt{5} + 2$. 1 mark for $-1 - \sqrt{5}$.
