

Solving Trigonometric Equations

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Solve $\sin x = 0.35$ for $0^\circ \leq x \leq 360^\circ$

Give your answers to 1 decimal place.

Calculator: $x = \sin^{-1} 0.35 = 20.48 \dots^\circ$

$\sin$ is positive in the 1st and 2nd quadrants.

The sine graph is symmetrical about $x = 90$, so the other solution is

$180 - 20.48 \dots = 159.51 \dots^\circ$

Answer: $x = 20.5^\circ$ and $x = 159.5^\circ$

Tip: The calculator gives only one answer. Use the graph or CAST to find the second solution in the range.

Marks: 1 mark for 20.5° . 1 mark for 159.5° .

2 (2 marks)

Solve $2 \cos x + \sqrt{3} = 0$ for $0^\circ \leq x \leq 360^\circ$

You must show your working.

Rearrange: $2 \cos x = -\sqrt{3}$, so $\cos x = -\frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, so the angle to use with the graph or CAST is 30° .

$\cos$ is negative in the 2nd and 3rd quadrants:

$$x = 180 - 30 = 150^\circ \text{ and } x = 180 + 30 = 210^\circ$$

Answer: $x = 150^\circ$ and $x = 210^\circ$

Tip: Learn the exact values: $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\cos 45^\circ = \frac{\sqrt{2}}{2}$. Then use the sign to decide the quadrants.

Marks: 1 mark for $\cos x = -\frac{\sqrt{3}}{2}$ and one correct angle. 1 mark for both angles (and no others).

3 (2 marks)

Solve $3 \sin x = 5 \cos x$ for $0^\circ \leq x \leq 360^\circ$

Give your answers to 1 decimal place.

Divide both sides by $\cos x$: $\frac{3 \sin x}{\cos x} = 5$

Use $\tan x = \frac{\sin x}{\cos x}$: $3 \tan x = 5$, so $\tan x = \frac{5}{3}$

Calculator: $x = \tan^{-1} \frac{5}{3} = 59.03 \dots^\circ$

$\tan$ repeats every 180° : $59.03 \dots + 180 = 239.03 \dots^\circ$

Answer: $x = 59.0^\circ$ and $x = 239.0^\circ$

Tip: Divide by $\cos x$ to get a single trig ratio. Make sure you get $\tan x = \frac{5}{3}$, not $\frac{3}{5}$: the number with $\sin x$ goes on the bottom.

Marks: 1 mark for $\tan x = \frac{5}{3}$. 1 mark for both answers.

4 (3 marks)

Solve $25 \sin^2 x = 16$ for $0^\circ \leq x \leq 360^\circ$

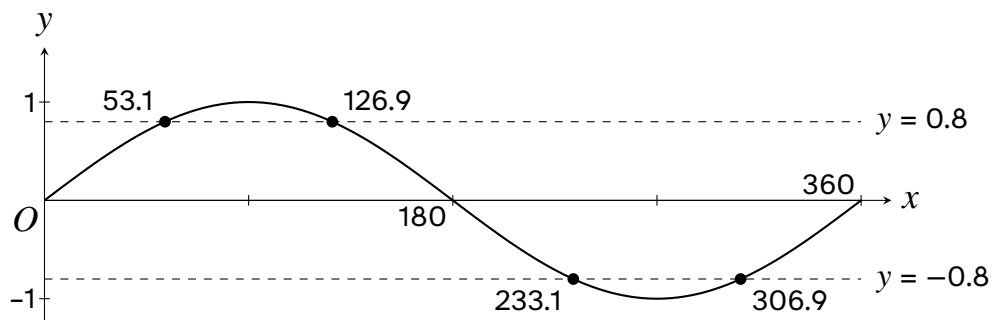
Give your answers to 1 decimal place.

$$\sin^2 x = \frac{16}{25}$$

Square root, remembering $\pm$: $\sin x = \frac{4}{5}$ or $\sin x = -\frac{4}{5}$

$\sin x = 0.8$: $x = \sin^{-1} 0.8 = 53.13 \dots^\circ$ and $180 - 53.13 \dots = 126.86 \dots^\circ$

$\sin x = -0.8$: $\sin$ is negative in the 3rd and 4th quadrants, so
 $x = 180 + 53.13 \dots = 233.13 \dots^\circ$ and $360 - 53.13 \dots = 306.86 \dots^\circ$



Answer: $x = 53.1^\circ, 126.9^\circ, 233.1^\circ, 306.9^\circ$

Tip: Don't forget the negative square root. In the exam many students used only + and found half of the solutions.

Marks: 1 mark for $\sin x = \pm 0.8$. 1 mark for two correct answers. 1 mark for all four (and no others).

5 (3 marks)

Solve $2 \sin x \cos x = \sin x$ for $0^\circ \leq x \leq 360^\circ$

Bring everything to one side: $2 \sin x \cos x - \sin x = 0$

Factorise: $\sin x (2 \cos x - 1) = 0$

So $\sin x = 0$ or $2 \cos x - 1 = 0$

$\sin x = 0$: $x = 0^\circ, 180^\circ, 360^\circ$

$\cos x = \frac{1}{2}$: $x = 60^\circ$ and $360 - 60 = 300^\circ$

Answer: $x = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$

Tip: Don't divide both sides by $\sin x$: that loses the solutions where $\sin x = 0$. Take out the common factor $\sin x$ instead.

Marks: 1 mark for factorising (or for $\sin x = 0$ and $\cos x = \frac{1}{2}$). 1 mark for 60° and 300° . 1 mark for $0^\circ, 180^\circ$ and 360° .

6 (3 marks)

Solve $\tan^2 x = 3$ for $90^\circ < x < 270^\circ$

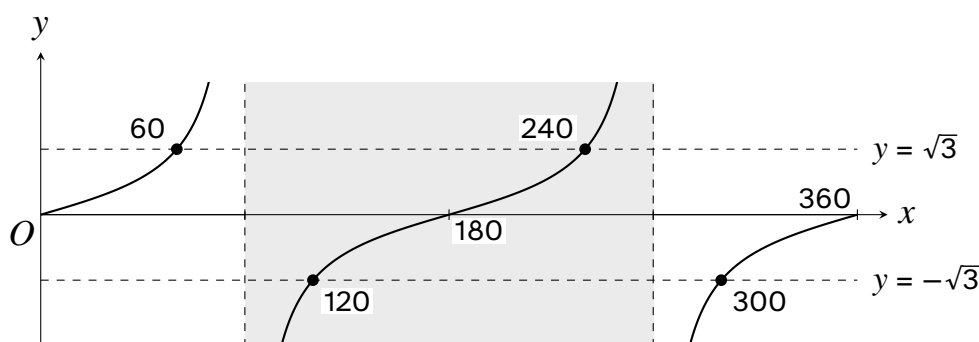
Square root, remembering $\pm$: $\tan x = \sqrt{3}$ or $\tan x = -\sqrt{3}$

$\tan 60^\circ = \sqrt{3}$, so the angle to use is 60° .

$\tan x = \sqrt{3}$ (tan positive, 1st and 3rd quadrants): $x = 60^\circ$ or 240°

$\tan x = -\sqrt{3}$ (tan negative, 2nd and 4th quadrants): $x = 120^\circ$ or 300°

Only 120° and 240° are between 90° and 270° .



Answer: $x = 120^\circ$ and $x = 240^\circ$

Tip: Find all the angles first, then cross out any outside the range given. Answers outside $90^\circ < x < 270^\circ$ lose the last mark.

Marks: 1 mark for $\tan x = \pm\sqrt{3}$. 1 mark for one correct angle. 1 mark for both, with no extra angles.

7 (4 marks)

Solve $3\cos^2 x + 5\sin x = 1$ for $0^\circ \leq x \leq 360^\circ$

Give your answers to 1 decimal place.

Use $\cos^2 x = 1 - \sin^2 x$ to get an equation in $\sin x$ only: $3(1 - \sin^2 x) + 5\sin x = 1$

$$3 - 3\sin^2 x + 5\sin x - 1 = 0$$

$$-3\sin^2 x + 5\sin x + 2 = 0, \text{ so } 3\sin^2 x - 5\sin x - 2 = 0$$

Factorise (like $3y^2 - 5y - 2 = (3y + 1)(y - 2)$): $(3\sin x + 1)(\sin x - 2) = 0$

$$\sin x = -\frac{1}{3} \text{ or } \sin x = 2$$

$\sin x = 2$ has no solutions, because $\sin x$ is never more than 1.

$\sin x = -\frac{1}{3}$: $\sin^{-1} \frac{1}{3} = 19.47 \dots^\circ$, and $\sin$ is negative in the 3rd and 4th quadrants:

$x = 180 + 19.47 \dots = 199.47 \dots^\circ$ and $x = 360 - 19.47 \dots = 340.52 \dots^\circ$

Answer: $x = 199.5^\circ$ and $x = 340.5^\circ$

Tip: Write $\cos^2 x = 1 - \sin^2 x$ in brackets, then treat it like a quadratic in $y = \sin x$. The calculator gives -19.5° for $\sin^{-1}(-\frac{1}{3})$, which is not in the range: use the graph or CAST.

Marks: 1 mark for replacing $\cos^2 x$ with $1 - \sin^2 x$. 1 mark for $3 \sin^2 x - 5 \sin x - 2 = 0$. 1 mark for $\sin x = -\frac{1}{3}$ (and rejecting $\sin x = 2$). 1 mark for both answers.

8 (4 marks)

- (a) Show that the equation $4 \sin x \tan x = 15$ can be written as $4 \cos^2 x + 15 \cos x - 4 = 0$

Use $\tan x = \frac{\sin x}{\cos x}$: $4 \sin x \times \frac{\sin x}{\cos x} = 15$, so $\frac{4 \sin^2 x}{\cos x} = 15$

Multiply both sides by $\cos x$: $4 \sin^2 x = 15 \cos x$

Use $\sin^2 x = 1 - \cos^2 x$: $4(1 - \cos^2 x) = 15 \cos x$

$4 - 4 \cos^2 x = 15 \cos x$

$0 = 4 \cos^2 x + 15 \cos x - 4$, so $4 \cos^2 x + 15 \cos x - 4 = 0$

Answer: $4 \sin^2 x = 15 \cos x$, so $4(1 - \cos^2 x) = 15 \cos x$, which gives $4 \cos^2 x + 15 \cos x - 4 = 0$

Tip: In a 'show that', write every step. Don't jump from the first line to the answer.

Marks: 1 mark for $4 \sin^2 x = 15 \cos x$. 1 mark for using $\sin^2 x = 1 - \cos^2 x$ and reaching the given equation.

- (b) Hence solve $4 \sin x \tan x = 15$ for $0^\circ \leq x \leq 360^\circ$
Give your answers to 1 decimal place.

Factorise: $(4 \cos x - 1)(\cos x + 4) = 0$

$$\cos x = \frac{1}{4} \text{ or } \cos x = -4$$

$\cos x = -4$ has no solutions ($\cos x$ is never less than -1).

$$\cos x = \frac{1}{4}: x = \cos^{-1} 0.25 = 75.52 \dots^\circ$$

The cosine graph is symmetrical about $x = 180$: $360 - 75.52 \dots = 284.47 \dots^\circ$

Answer: $x = 75.5^\circ$ and $x = 284.5^\circ$

Tip: Check each value from the quadratic: $\cos x = -4$ is impossible, so it gives no angles. Write that down rather than leaving it out silently.

Marks: 1 mark for $\cos x = \frac{1}{4}$. 1 mark for both answers.
