

Sketching Graphs and Exponential Graphs

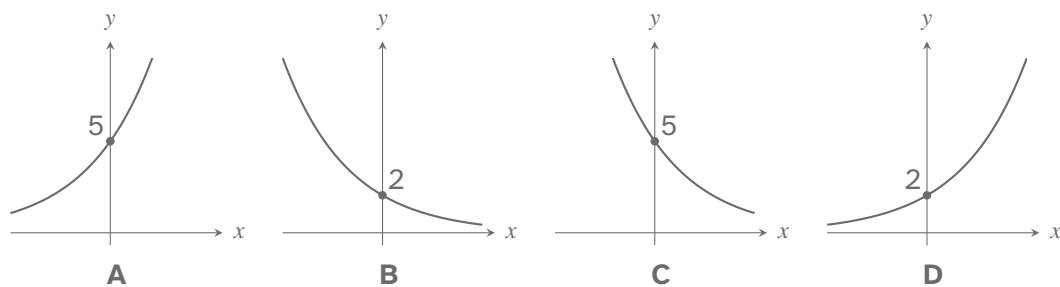
Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here are four sketches.



Which sketch shows the graph of $y = 5 \times 2^{-x}$?

Circle your answer.

Options: A, B, C, D

y-intercept: put $x = 0$. $2^0 = 1$, so $y = 5 \times 1 = 5$

That leaves A or C.

$2^{-x} = \left(\frac{1}{2}\right)^x$, which gets smaller as x increases.

So the curve goes down from left to right: sketch C.

Answer: C

Tip: Check the y-intercept first: for $y = a \times b^x$ or $y = a \times b^{-x}$ it is always a . The minus sign in the power makes the curve go down from left to right.

2 (1 mark)

A quadratic curve has a turning point at $(3, -2)$

Which of these could be its equation?

Circle your answer.

Options: $y = (x + 3)^2 - 2$, $y = (x - 3)^2 + 2$, $y = (x - 3)^2 - 2$, $y = (x + 2)^2 - 3$

$y = (x - p)^2 + q$ has its turning point at (p, q) .

$(x - 3)^2$ is smallest (zero) when $x = 3$, and then $y = -2$.

So $y = (x - 3)^2 - 2$ has turning point $(3, -2)$.

Answer: $y = (x - 3)^2 - 2$

Tip: The sign inside the bracket is the opposite of the x -coordinate: $x = 3$ goes with $(x - 3)^2$.

3 (3 marks)

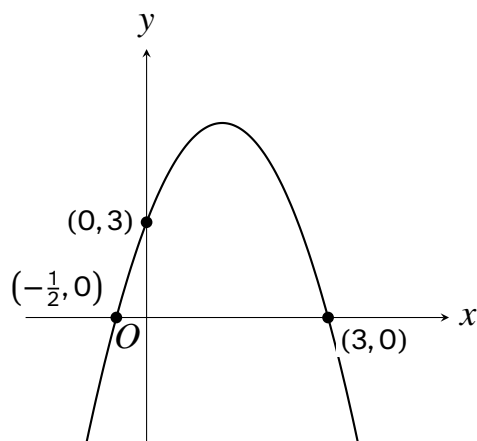
Sketch the graph of $y = (3 - x)(2x + 1)$. Label the coordinates of the points where the graph meets the axes.

Multiplying out, the x^2 term is $-x \times 2x = -2x^2$. This is negative, so the curve is an upside-down U (a $\cap$ shape).

x -axis: $y = 0$ when $3 - x = 0$ or $2x + 1 = 0$

So $x = 3$ or $x = -\frac{1}{2}$: the points $(3, 0)$ and $(-\frac{1}{2}, 0)$

y -axis: $x = 0$ gives $y = 3 \times 1 = 3$: the point $(0, 3)$



Answer: $\cap$ -shaped curve through $(-\frac{1}{2}, 0)$, $(3, 0)$ and $(0, 3)$

Tip: Look at the sign of the x^2 term before you draw: $-2x^2$ means the curve opens downwards.
The root from $(2x + 1)$ is $x = -\frac{1}{2}$, not $x = 1$ or $x = -1$.

Marks: 1 mark for a $\cap$ shape. 1 mark for both x -intercepts. 1 mark for the y -intercept.

4 (3 marks)

Sketch the graph of $y = (x + 1)(x - 2)^2$. Label the coordinates of the points where the graph meets the axes.

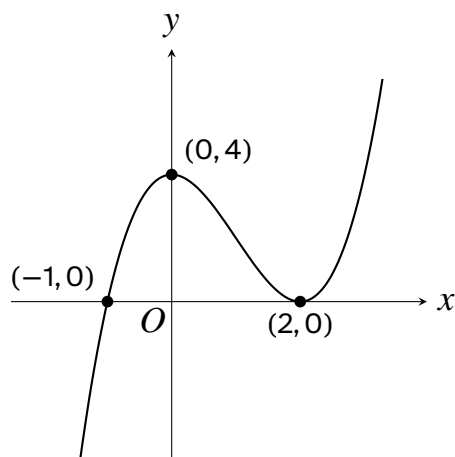
The x^3 term is $+x^3$, so the curve goes from bottom left to top right.

x -axis: $y = 0$ when $x = -1$ or $x = 2$

$(x - 2)$ is squared, so $x = 2$ is a repeated root: the curve **touches** the x -axis at $(2, 0)$ and turns back up.

At $x = -1$ the curve crosses the x -axis.

y -axis: $x = 0$ gives $y = 1 \times (-2)^2 = 4$: the point $(0, 4)$



Answer: Cubic crossing at $(-1, 0)$, touching at $(2, 0)$, through $(0, 4)$

Tip: A squared factor gives a repeated root, where the curve just touches the axis. Drawing it crossing at $x = 2$ loses a mark.

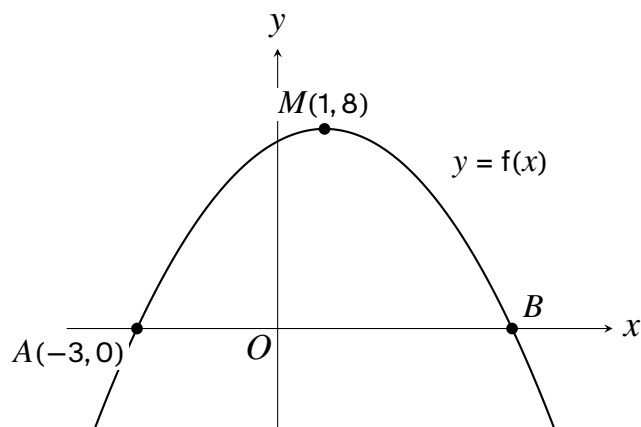
Marks: 1 mark for a cubic shape that crosses at one point and touches at another. 1 mark for $(-1, 0)$ and $(2, 0)$. 1 mark for $(0, 4)$.

5 (5 marks)

The sketch shows the graph of the quadratic function $y = f(x)$

The graph crosses the x -axis at $A(-3, 0)$ and at B .

Its highest point is $M(1, 8)$



Not drawn accurately

- (a) Write down the coordinates of B .

A quadratic curve is symmetrical about the vertical line through its turning point, $x = 1$.

A is $1 - (-3) = 4$ units to the left of this line.

So B is 4 units to the right: $x = 1 + 4 = 5$

Answer: (5, 0)

Tip: The turning point is exactly halfway between the two roots. Find how far A is from the line $x = 1$, then go the same distance on the other side.

- (b) Work out $f(x)$

Give your answer in the form $ax^2 + bx + c$

The roots are $x = -3$ and $x = 5$, so $f(x) = a(x + 3)(x - 5)$ for some number a .

$M(1, 8)$ is on the curve, so $f(1) = 8$:

$$a(1 + 3)(1 - 5) = 8$$

$$-16a = 8, \text{ so } a = -\frac{1}{2}$$

$$f(x) = -\frac{1}{2}(x + 3)(x - 5) = -\frac{1}{2}(x^2 - 2x - 15)$$

$$= -\frac{1}{2}x^2 + x + \frac{15}{2}$$

Answer: $f(x) = -\frac{1}{2}x^2 + x + \frac{15}{2}$

Tip: Don't assume the x^2 coefficient is 1 (or -1). Use the turning point to find it. Check:

$$f(1) = -\frac{1}{2} + 1 + \frac{15}{2} = 8.$$

Marks: 1 mark for $a(x+3)(x-5)$. 1 mark for $a = -\frac{1}{2}$. 1 mark for the expanded answer.

(c) The line $y = k$ does not meet the curve.

Write down the possible values of k .

Give your answer as an inequality.

$y = k$ is a horizontal line.

The highest point of the curve is $M(1, 8)$.

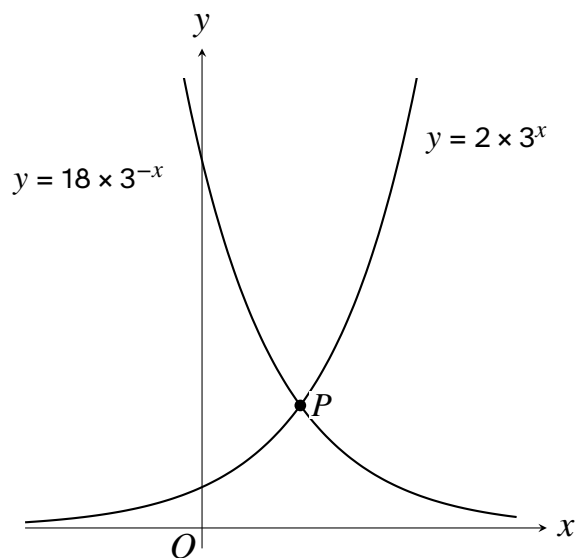
The line misses the curve completely if it is above M : $k > 8$

Answer: $k > 8$

Tip: If $k = 8$ the line touches the curve at M , so it does meet it. That is why the inequality is strict.

6 (4 marks)

The sketch shows the curves $y = 2 \times 3^x$ and $y = 18 \times 3^{-x}$
The curves intersect at P .



Not drawn accurately

- (a) Write down the coordinates of the point where the curve $y = 18 \times 3^{-x}$ crosses the y-axis.

On the y-axis, $x = 0$.

$$3^0 = 1, \text{ so } y = 18 \times 1 = 18$$

Answer: (0,18)

- (b) Work out the coordinates of P .
You must show your working.

$$\text{At } P \text{ the two } y\text{-values are equal: } 2 \times 3^x = 18 \times 3^{-x}$$

$$\text{Multiply both sides by } 3^x: 2 \times 3^{2x} = 18$$

$$\text{Divide by 2: } 3^{2x} = 9 = 3^2$$

$$\text{Equate the powers: } 2x = 2, \text{ so } x = 1$$

$$y = 2 \times 3^1 = 6$$

Check: $18 \times 3^{-1} = 18 \div 3 = 6$ ✓

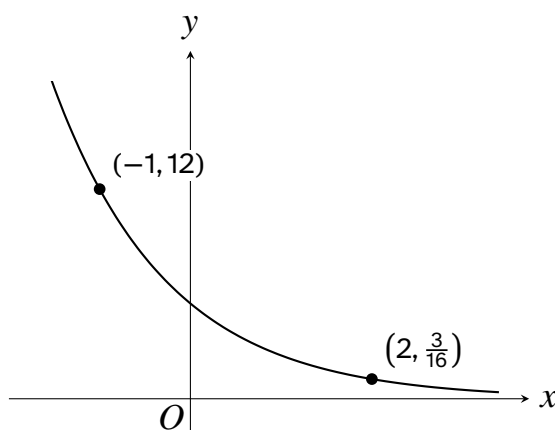
Answer: (1, 6)

Tip: Write both sides as powers of 3, then equate the powers. 2×3^x means $2 \times (3^x)$, not 6^x .

Marks: 1 mark for $2 \times 3^x = 18 \times 3^{-x}$. 1 mark for $3^{2x} = 9$ (or equivalent). 1 mark for (1, 6).

7 (6 marks)

The graph of $y = a \times b^{-x}$ goes through the points $(-1, 12)$ and $(2, \frac{3}{16})$
 $a > 0$ and $b > 0$



Not drawn accurately

(a) Work out the value of a and the value of b .

Substitute $(-1, 12)$: $a \times b^{-(-1)} = 12$, so $ab = 12$ (1)

Substitute $(2, \frac{3}{16})$: $a \times b^{-2} = \frac{3}{16}$, so $\frac{a}{b^2} = \frac{3}{16}$ (2)

Divide (1) by (2): $ab \div \frac{a}{b^2} = b^3$

$$b^3 = 12 \div \frac{3}{16} = 12 \times \frac{16}{3} = 64$$

$$b = \sqrt[3]{64} = 4$$

Substitute into (1): $4a = 12$, so $a = 3$

Check (2): $3 \times 4^{-2} = \frac{3}{16}$ ✓

Answer: $a = 3, b = 4$

Tip: Take care with the double minus: when $x = -1$, $b^{-x} = b^1 = b$. Dividing one equation by the other cancels a and leaves one power of b .

Marks: 1 mark for one correct equation. 1 mark for both equations. 1 mark for $b = 4$. 1 mark for $a = 3$.

(b) Work out the value of x when $y = 48$

Use $a = 3$ and $b = 4$: $3 \times 4^{-x} = 48$

Divide by 3: $4^{-x} = 16$

$16 = 4^2$, so $4^{-x} = 4^2$

Equate the powers: $-x = 2$, so $x = -2$

Check: $3 \times 4^{-(-2)} = 3 \times 4^2 = 48$ ✓

Answer: $x = -2$

Tip: Divide by a first: 3×4^{-x} is not 12^{-x} . A negative answer makes sense here: the curve is higher on the left.

Marks: 1 mark for $4^{-x} = 16$. 1 mark for $x = -2$.

8 (4 marks)

Here is the graph of $y = x^2 - x - 3$ for values of x from -2 to 4

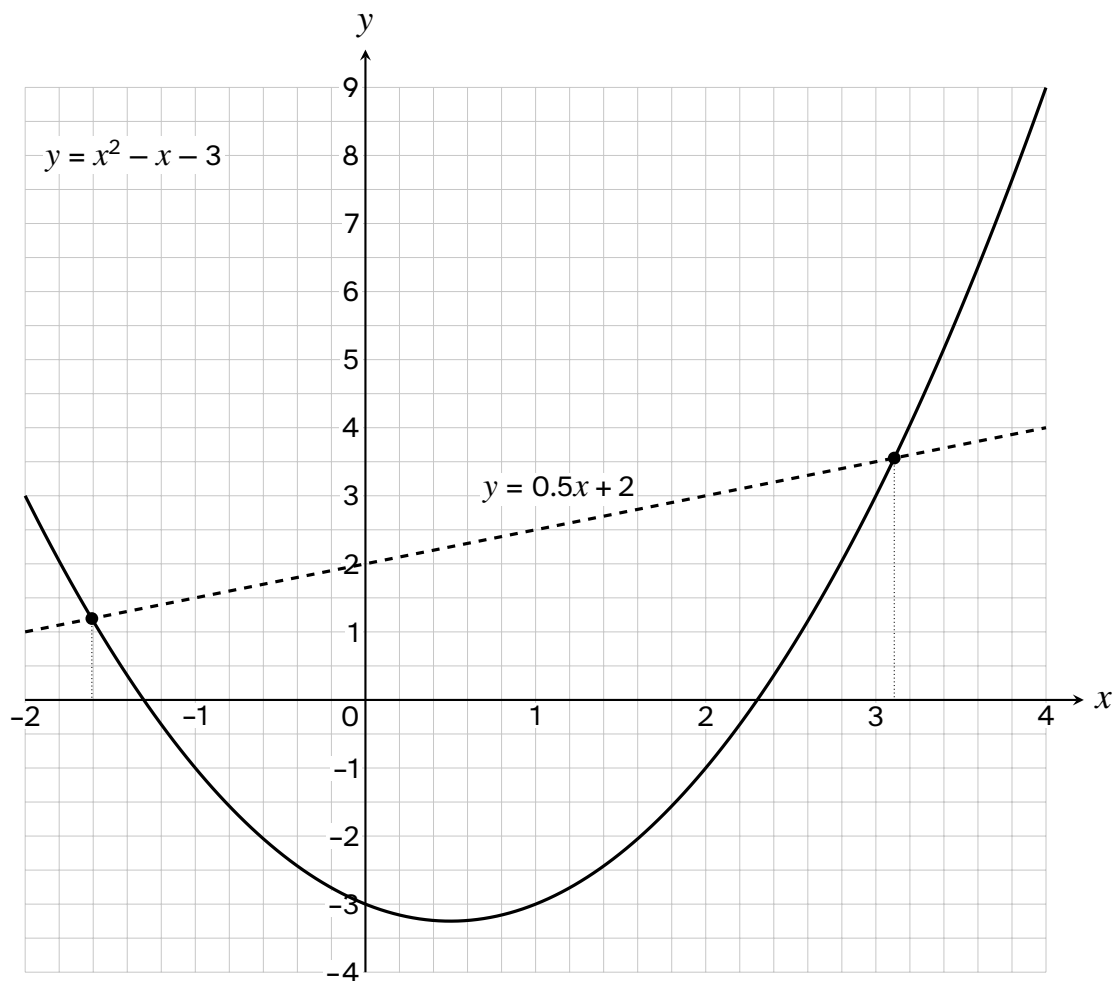
By drawing a suitable straight line on the grid, solve $2x^2 - 3x - 10 = 0$
Give your answers to 1 decimal place.

Divide the equation by 2 so that it starts with x^2 : $x^2 - \frac{3}{2}x - 5 = 0$

Rearrange so that $x^2 - x - 3$ (the curve) is on the left:

Add $\frac{1}{2}x$ to both sides and add 2 to both sides: $x^2 - x - 3 = \frac{1}{2}x + 2$

So draw the line $y = 0.5x + 2$. It passes through $(-2, 1)$, $(0, 2)$ and $(4, 4)$.



Read the x -values where the line crosses the curve: $x \approx -1.6$ and $x \approx 3.1$

Answers from -1.5 to -1.7 and from 3.0 to 3.2 are fine if read from an accurate graph.

Answer: $x \approx -1.6$ and $x \approx 3.1$

Tip: This was very poorly answered in the exam. Get the curve's expression, $x^2 - x - 3$, on one side; the other side is the line to draw. Don't differentiate, and read the x -values carefully, watching the signs.

Marks: 1 mark for rearranging to get $x^2 - x - 3 = 0.5x + 2$. 1 mark for the line $y = 0.5x + 2$ drawn correctly. 1 mark for each solution.