

Sine Rule, Cosine Rule and Area of a Triangle

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the value of $\sin 150^\circ$

Options: $-\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$

The sine graph is symmetrical about 90° , so $\sin(180^\circ - x) = \sin x$

$$\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ$$

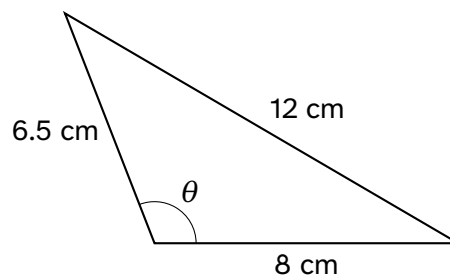
$$\sin 30^\circ = \frac{1}{2}$$

Answer: $\frac{1}{2}$

Tip: Sine is positive for every angle between 0° and 180° , so the answer can't be negative. This fact is used whenever a triangle has an obtuse angle.

2 (3 marks)

Here is a triangle.



Not drawn accurately

Work out the size of angle θ .

Give your answer to 1 decimal place.

θ is opposite the longest side, 12 cm.

$$\text{Cosine rule: } 12^2 = 6.5^2 + 8^2 - 2 \times 6.5 \times 8 \cos \theta$$

$$144 = 106.25 - 104 \cos \theta$$

$$104 \cos \theta = 106.25 - 144 = -37.75$$

$$\cos \theta = -\frac{37.75}{104} = -0.3629 \dots$$

$$\theta = \cos^{-1}(-0.3629 \dots) = 111.28 \dots$$

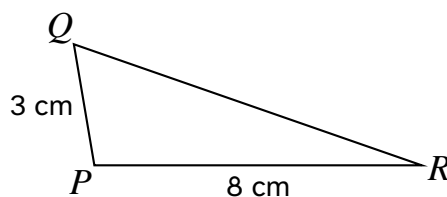
Answer: $\theta = 111.3^\circ$

Tip: A negative cosine means the angle is obtuse. That fits here: $12^2 = 144$ is more than $6.5^2 + 8^2 = 106.25$, so the angle opposite 12 cm must be more than 90° .

Marks: 1 mark for substituting into the cosine rule. 1 mark for $\cos \theta = -\frac{37.75}{104}$ (or -0.363). 1 mark for 111.3.

3 (3 marks)

Here is triangle PQR .



Not drawn accurately

$$\cos QPR = -\frac{1}{6}$$

Work out the length QR .

You must show your working.

Use the given value of $\cos QPR$ straight in the cosine rule. There is no need to find the angle.

$$QR^2 = 3^2 + 8^2 - 2 \times 3 \times 8 \times \left(-\frac{1}{6}\right)$$

$$2 \times 3 \times 8 \times \left(-\frac{1}{6}\right) = 48 \times \left(-\frac{1}{6}\right) = -8$$

$$QR^2 = 9 + 64 - (-8) = 73 + 8 = 81$$

$$QR = \sqrt{81} = 9$$

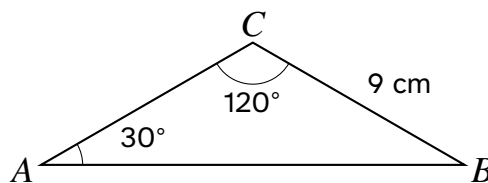
Answer: 9 cm

Tip: Work out $2bc \cos A$ first, then subtract it. Subtracting -8 means adding 8.

Marks: 1 mark for a correct substitution. 1 mark for $QR^2 = 81$. 1 mark for 9.

4 (3 marks)

Here is triangle ABC .



Not drawn accurately

Work out the exact length of AB .

Give your answer in the form $a\sqrt{3}$, where a is an integer.

AB is opposite the 120° angle and 9 cm is opposite the 30° angle.

$$\text{Sine rule: } \frac{AB}{\sin 120^\circ} = \frac{9}{\sin 30^\circ}$$

$$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin 30^\circ = \frac{1}{2}$$

$$AB = \frac{9 \sin 120^\circ}{\sin 30^\circ} = 9 \times \frac{\sqrt{3}}{2} \div \frac{1}{2}$$

$$= 9 \times \frac{\sqrt{3}}{2} \times 2 = 9\sqrt{3}$$

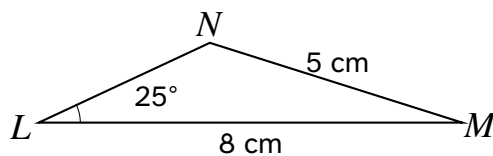
Answer: $9\sqrt{3}\text{ cm}$

Tip: $\sin 120^\circ$ is not $\sin 90^\circ + \sin 30^\circ$, and it isn't negative. Use $\sin(180^\circ - x) = \sin x$. Then take care dividing by the fraction $\frac{1}{2}$: it doubles the number.

Marks: 1 mark for the sine rule with the angles paired correctly. 1 mark for $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\sin 30^\circ = \frac{1}{2}$. 1 mark for $9\sqrt{3}$.

5 (4 marks)

Here is triangle LMN .



Not drawn accurately

Angle LMN is obtuse.

Work out the area of triangle LMN .

Step 1: find angle LMN . 8 cm is opposite angle N and 5 cm is opposite the 25° angle.

Sine rule: $\frac{\sin N}{8} = \frac{\sin 25^\circ}{5}$, so $\sin N = \frac{8 \sin 25^\circ}{5} = 0.6761 \dots$

Calculator: $\sin^{-1}(0.6761 \dots) = 42.54 \dots^\circ$, which is acute.

$\sin(180^\circ - x) = \sin x$, so angle $LMN = 180^\circ - 42.54 \dots^\circ = 137.45 \dots^\circ$

Step 2: angle $LMN = 180^\circ - 25^\circ - 137.45 \dots^\circ = 17.54 \dots^\circ$

This is the angle between the 8 cm and 5 cm sides.

Step 3: area = $\frac{1}{2} \times 8 \times 5 \times \sin 17.54 \dots^\circ = 6.029 \dots$

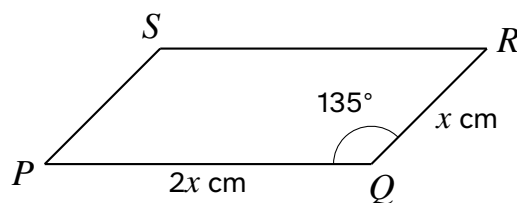
Answer: 6.03 cm²

Tip: Your calculator only ever gives the acute angle from $\sin^{-1}$. When the angle is obtuse, take your answer away from 180° . Using 42.5° here gives a different (wrong) triangle.

Marks: 1 mark for $\sin N = \frac{8 \sin 25^\circ}{5}$. 1 mark for 137.5° . 1 mark for angle $M = 17.5^\circ$ and $\frac{1}{2} \times 8 \times 5 \sin M$. 1 mark for 6.03.

6 (3 marks)

$PQRS$ is a parallelogram.



Not drawn accurately

The area of the parallelogram is $36\sqrt{2} \text{ cm}^2$

Work out the value of x .

You must show your working.

A diagonal splits the parallelogram into two identical triangles, so the area is

$$2 \times \frac{1}{2}ab \sin C = ab \sin C$$

$$\text{Area} = 2x \times x \times \sin 135^\circ = 2x^2 \sin 135^\circ$$

$$\sin 135^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\text{Area} = 2x^2 \times \frac{\sqrt{2}}{2} = \sqrt{2}x^2$$

$$\sqrt{2}x^2 = 36\sqrt{2}, \text{ so } x^2 = 36$$

x is a length, so $x = 6$

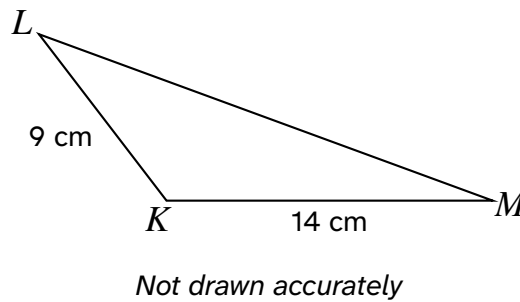
Answer: $x = 6$

Tip: $\sin 135^\circ$ is not $\sin 90^\circ + \sin 45^\circ$. Use the symmetry of the sine graph: $\sin 135^\circ = \sin 45^\circ$.

Marks: 1 mark for $2x \times x \times \sin 135^\circ$ (or $\frac{1}{2} \times 2x \times x \sin 135^\circ$ doubled). 1 mark for $\sin 135^\circ = \frac{\sqrt{2}}{2}$. 1 mark for 6.

7 (4 marks)

Here is triangle KLM .



$KL = 9$ cm and $KM = 14$ cm

The area of triangle KLM is 50 cm²

Angle LKM is obtuse.

Work out the length LM .

Step 1: use the area to find angle LKM .

$$\frac{1}{2} \times 9 \times 14 \times \sin K = 50$$

$$63 \sin K = 50, \text{ so } \sin K = \frac{50}{63} = 0.7936 \dots$$

$$\sin^{-1}(0.7936 \dots) = 52.52 \dots^\circ, \text{ but angle } LKM \text{ is obtuse.}$$

$$\text{Angle } LKM = 180^\circ - 52.52 \dots^\circ = 127.47 \dots^\circ$$

Step 2: cosine rule.

$$LM^2 = 9^2 + 14^2 - 2 \times 9 \times 14 \cos 127.47 \dots^\circ$$

$$LM^2 = 277 + 153.31 \dots = 430.31 \dots$$

$$LM = 20.74 \dots$$

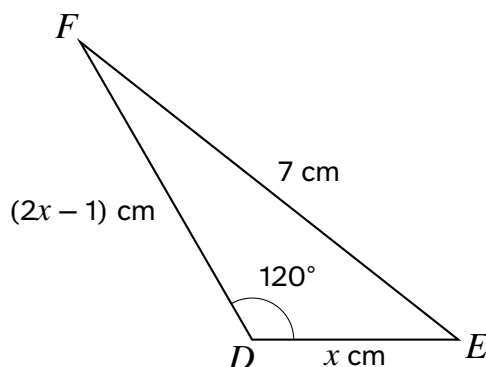
Answer: 20.7 cm

Tip: Using the acute angle 52.5° gives 11.1 cm, which is wrong here. Read the question for clues about which angle you need.

Marks: 1 mark for $\frac{1}{2} \times 9 \times 14 \sin K = 50$. 1 mark for 127.5° . 1 mark for a correct cosine rule. 1 mark for 20.7.

8 (6 marks)

Here is triangle DEF .



Not drawn accurately

- (a) Show that $7x^2 - 5x - 48 = 0$

The 7 cm side is opposite the 120° angle.

Cosine rule: $7^2 = x^2 + (2x - 1)^2 - 2x(2x - 1) \cos 120^\circ$

$\cos 120^\circ = -\frac{1}{2}$, so $-2x(2x - 1) \cos 120^\circ = +x(2x - 1)$

$$49 = x^2 + 4x^2 - 4x + 1 + 2x^2 - x$$

$$49 = 7x^2 - 5x + 1$$

$$7x^2 - 5x - 48 = 0 \quad \checkmark$$

Answer: Shown: $7x^2 - 5x - 48 = 0$

Tip: Put brackets round $(2x - 1)$ before squaring it: $(2x - 1)^2 = 4x^2 - 4x + 1$, not $4x^2 + 1$ or $2x^2 - 1$.

Marks: 1 mark for a correct cosine rule with $(2x - 1)$ in brackets. 1 mark for using $\cos 120^\circ = -\frac{1}{2}$. 1 mark for a correct expansion leading to the given equation.

- (b) Work out the area of triangle DEF .

Give your answer in the form $\frac{a\sqrt{3}}{b}$, where a and b are integers.

Factorise: $(7x + 16)(x - 3) = 0$, so $x = -\frac{16}{7}$ or $x = 3$

x is a length, so $x = 3$. The sides are $DE = 3$ cm and $DF = 2 \times 3 - 1 = 5$ cm.

$$\text{Area} = \frac{1}{2} \times 3 \times 5 \times \sin 120^\circ$$

$$\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Area} = \frac{15}{2} \times \frac{\sqrt{3}}{2} = \frac{15\sqrt{3}}{4}$$

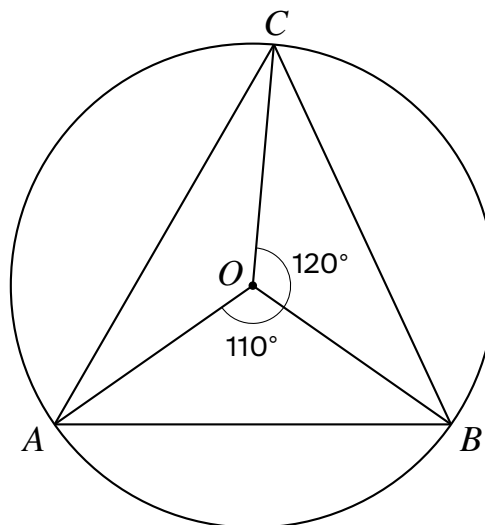
Answer: $\frac{15\sqrt{3}}{4} \text{ cm}^2$

Tip: Reject $x = -\frac{16}{7}$: a length can't be negative. Use the sides x and $2x - 1$ (3 and 5), which are either side of the 120° angle.

Marks: 1 mark for $x = 3$. 1 mark for $\frac{1}{2} \times 3 \times 5 \times \sin 120^\circ$. 1 mark for $\frac{15\sqrt{3}}{4}$.

9 (4 marks)

The diagram shows a triangle inside a circle.



Not drawn accurately

A , B and C are points on a circle, centre O , radius 8 cm.

Angle $AOB = 110^\circ$ and angle $BOC = 120^\circ$

Work out the area of triangle ABC .

Angles around a point add up to 360° : angle $AOC = 360^\circ - 110^\circ - 120^\circ = 130^\circ$

O is inside triangle ABC , so the area of ABC is the sum of the areas of triangles AOB , BOC and COA .

Each of these has two sides of 8 cm (radii) with the angle at O between them.

$$AOB: \frac{1}{2} \times 8 \times 8 \times \sin 110^\circ = 30.07 \dots$$

$$BOC: \frac{1}{2} \times 8 \times 8 \times \sin 120^\circ = 27.71 \dots$$

$$COA: \frac{1}{2} \times 8 \times 8 \times \sin 130^\circ = 24.51 \dots$$

$$\text{Total: } 30.07 \dots + 27.71 \dots + 24.51 \dots = 82.29 \dots$$

Answer: 82.3 cm²

Tip: Don't forget the third triangle, COA : its angle comes from angles around a point. Keep the unrounded areas and round only at the end.

Marks: 1 mark for angle $AOC = 130^\circ$. 1 mark for $\frac{1}{2} \times 8 \times 8 \times \sin$ of one of the angles. 1 mark for adding all three areas. 1 mark for 82.3.
