

Simultaneous Equations (Two Unknowns)

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the point that lies on both $y = x^2 - 1$ and $y = x + 5$

Options: (2, 3), (3, 8), (8, 3), (-3, 2)

A point on both graphs must fit **both** equations.

(2, 3): $2^2 - 1 = 3$ fits the curve, but $2 + 5 = 7 \neq 3$, so it is not on the line.

(3, 8): $3^2 - 1 = 8$ and $3 + 5 = 8$. It fits both.

(8, 3): $8^2 - 1 = 63 \neq 3$. (-3, 2): $-3 + 5 = 2$ fits the line, but $(-3)^2 - 1 = 8 \neq 2$.

Answer: (3, 8)

Tip: Check each point in **both** equations. Two of the wrong options fit one equation but not the other.

2 (3 marks)

Solve the simultaneous equations

$$\frac{x}{3} + \frac{y}{2} = 4$$

$$0.5x - 0.2y = 2.2$$

Clear the fractions and decimals first.

Multiply the first equation by 6: $2x + 3y = 24$ (1)

Multiply the second equation by 10: $5x - 2y = 22$ (2)

Make the y terms match: (1)×2: $4x + 6y = 48$ (2)×3: $15x - 6y = 66$

Add: $19x = 114$, so $x = 6$

Substitute into (1): $12 + 3y = 24$, so $3y = 12$, $y = 4$

(Check in (2): $5 \times 6 - 2 \times 4 = 30 - 8 = 22$ correct)

Answer: $x = 6$, $y = 4$

Tip: Multiply **every** term when you clear fractions: $\frac{x}{3} + \frac{y}{2} = 4$ times 6 is $2x + 3y = 24$, not $2x + 3y = 4$.

Marks: 1 mark for clearing the fractions and decimals correctly. 1 mark for a correct method to eliminate one letter. 1 mark for both values.

3 (3 marks)

k is a constant.

Solve the simultaneous equations

$$3x - y = 5k + 1$$

$$x + 2y = 4k - 2$$

Give x and y in terms of k .

Treat k like a number. Eliminate y :

Multiply the first equation by 2: $6x - 2y = 10k + 2$

Add the second equation: $(6x - 2y) + (x + 2y) = (10k + 2) + (4k - 2)$

$$7x = 14k, \text{ so } x = 2k$$

Substitute into the first equation: $3(2k) - y = 5k + 1$

$$6k - y = 5k + 1, \text{ so } y = 6k - 5k - 1 = k - 1$$

(Check in the second equation: $2k + 2(k - 1) = 4k - 2$ correct)

Answer: $x = 2k$, $y = k - 1$

Tip: You are solving for x and y , not k . Collect the k terms carefully: $6k - 5k - 1$ is $k - 1$.

Marks: 1 mark for eliminating x or y correctly. 1 mark for $x = 2k$. 1 mark for $y = k - 1$.

4 (4 marks)

Solve the simultaneous equations

$$x - 2y = 1$$

$$x^2 - 2y^2 = 17$$

You must show your working.

Make x the subject of the linear equation: $x = 2y + 1$

Substitute into the second equation: $(2y + 1)^2 - 2y^2 = 17$

Expand: $(2y + 1)^2 = 4y^2 + 4y + 1$

$$4y^2 + 4y + 1 - 2y^2 = 17$$

$$2y^2 + 4y - 16 = 0$$

Divide by 2: $y^2 + 2y - 8 = 0$

Factorise: $(y + 4)(y - 2) = 0$

So $y = 2$ or $y = -4$

Use $x = 2y + 1$: $y = 2$ gives $x = 5$; $y = -4$ gives $x = -7$

Answer: $x = 5, y = 2$ and $x = -7, y = -4$

Tip: Substitute from the **linear** equation, and square the whole bracket: $(2y + 1)^2 = 4y^2 + 4y + 1$, not $2y^2 + 1$.

Marks: 1 mark for substituting $x = 2y + 1$. 1 mark for $y^2 + 2y - 8 = 0$ (or $2y^2 + 4y - 16 = 0$). 1 mark for both y values. 1 mark for both pairs.

5 (5 marks)

Solve the simultaneous equations

$$xy = 2$$

$$3x + 4y = 10$$

Do not use trial and improvement.

You must show your working.

Make x the subject of the linear equation: $x = \frac{10 - 4y}{3}$

Substitute into $xy = 2$: $\frac{(10 - 4y)y}{3} = 2$

Multiply both sides by 3: $10y - 4y^2 = 6$

Rearrange: $4y^2 - 10y + 6 = 0$

Divide by 2: $2y^2 - 5y + 3 = 0$

Factorise: $(2y - 3)(y - 1) = 0$

So $y = 1$ or $y = \frac{3}{2}$

Use $x = \frac{2}{y}$: $y = 1$ gives $x = 2$; $y = \frac{3}{2}$ gives $x = 2 \div \frac{3}{2} = \frac{4}{3}$

(Check: $3 \times \frac{4}{3} + 4 \times \frac{3}{2} = 4 + 6 = 10$ correct)

Answer: $x = 2, y = 1$ and $x = \frac{4}{3}, y = \frac{3}{2}$

Tip: Clear the fraction before you try to factorise: multiply by 3 to get whole-number coefficients. Then give x and y in pairs; four numbers with no pairing loses the last mark.

Marks: 1 mark for substituting to get an equation in one letter. 1 mark for a quadratic with whole-number coefficients, e.g. $2y^2 - 5y + 3 = 0$. 1 mark for factorising or using the formula correctly. 1 mark for both y values. 1 mark for both pairs.

6 (4 marks)

A curve has equation $y = ax^2 + bx - 3$, where a and b are constants.

The curve passes through the point $(-1, 6)$.

The gradient of the curve at the point where $x = 2$ is 1.

Work out the values of a and b .

The point $(-1, 6)$ is on the curve, so put $x = -1, y = 6$ into the equation:

$$6 = a(-1)^2 + b(-1) - 3, \text{ so } a - b = 9 \quad (1)$$

Differentiate to find the gradient: $\frac{dy}{dx} = 2ax + b$

$$\text{When } x = 2 \text{ the gradient is 1: } 4a + b = 1 \quad (2)$$

$$\text{Add (1) and (2): } 5a = 10, \text{ so } a = 2$$

$$\text{Substitute into (1): } 2 - b = 9, \text{ so } b = -7$$

(Check: $y = 2x^2 - 7x - 3$ gives $2 + 7 - 3 = 6$ at $x = -1$, and $\frac{dy}{dx} = 4x - 7 = 1$ at $x = 2$ correct)

Answer: $a = 2, b = -7$

Tip: The point gives one equation and the gradient gives another. Don't put the point into $\frac{dy}{dx}$: the gradient is 1 at $x = 2$, not at $(-1, 6)$.

Marks: 1 mark for $a - b - 3 = 6$ or $a - b = 9$. 1 mark for $\frac{dy}{dx} = 2ax + b$. 1 mark for $4a + b = 1$. 1 mark for both values.

7 (5 marks)

Two squares have a total area of 106 cm^2 .

The perimeter of the larger square is 16 cm more than the perimeter of the smaller square.

Work out the side length of each square.

You must show your working.

Let the larger square have side a cm and the smaller square side b cm.

Total area: $a^2 + b^2 = 106$

Perimeters: $4a = 4b + 16$, so $a = b + 4$

Substitute: $(b + 4)^2 + b^2 = 106$

Expand: $b^2 + 8b + 16 + b^2 = 106$

$$2b^2 + 8b - 90 = 0$$

Divide by 2: $b^2 + 4b - 45 = 0$

Factorise: $(b + 9)(b - 5) = 0$

So $b = 5$ or $b = -9$. A length can't be negative, so $b = 5$.

Then $a = 5 + 4 = 9$

(Check: $81 + 25 = 106$ correct)

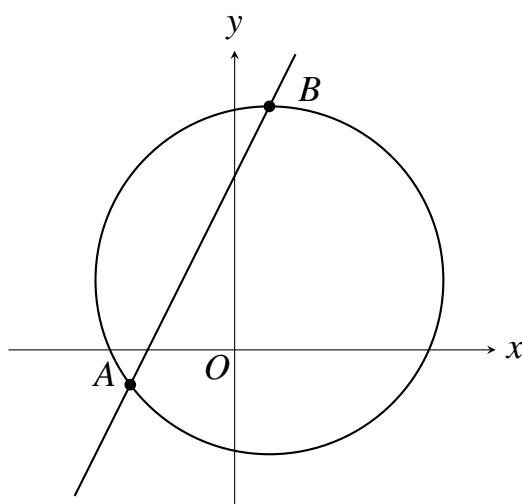
Answer: Larger square: 9 cm. Smaller square: 5 cm

Tip: Read “16 cm more” carefully: the **perimeters** differ by 16, so the sides differ by 4, not 16. Reject the negative solution and say why.

Marks: 1 mark for $a^2 + b^2 = 106$. 1 mark for $a = b + 4$ (or $4a - 4b = 16$). 1 mark for a correct quadratic in one letter. 1 mark for $b = 5$ (with -9 rejected). 1 mark for both sides.

8 (5 marks)

The circle $(x - 1)^2 + (y - 2)^2 = 25$ and the line $y = 2x + 5$ meet at the points A and B .



Not drawn accurately

Work out the coordinates of A and B .

You must show your working.

At A and B both equations are true, so substitute $y = 2x + 5$ into the circle:

$$(x - 1)^2 + (2x + 5 - 2)^2 = 25$$

$$(x - 1)^2 + (2x + 3)^2 = 25$$

$$\text{Expand: } x^2 - 2x + 1 + 4x^2 + 12x + 9 = 25$$

$$5x^2 + 10x + 10 = 25$$

$$5x^2 + 10x - 15 = 0$$

$$\text{Divide by 5: } x^2 + 2x - 3 = 0$$

Factorise: $(x + 3)(x - 1) = 0$, so $x = -3$ or $x = 1$

Find y from the line: $x = -3$ gives $y = -6 + 5 = -1$; $x = 1$ gives $y = 2 + 5 = 7$

From the diagram, A is the lower point and B is the higher point.

Answer: $A(-3, -1)$, $B(1, 7)$

Tip: Write $(2x + 3)^2$ as $(2x + 3)(2x + 3) = 4x^2 + 12x + 9$. Squaring each term separately ($4x^2 + 9$) is the most common error. Reading the points off the diagram is not a method.

Marks: 1 mark for substituting the line into the circle. 1 mark for expanding the brackets correctly. 1 mark for $x^2 + 2x - 3 = 0$ (or any multiple). 1 mark for both x values. 1 mark for both points, correctly labelled.
