

Simultaneous Equations (Three Unknowns)

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Here are three simultaneous equations.

$$x + y + z = 6$$

$$2x - y + z = 3$$

$$x + 2y - z = 2$$

Circle the solution, written as (x, y, z) .

Options: $(3, 3, 0)$, $(4, 0, 2)$, $(1, 2, 3)$, $(2, -1, -2)$

The solution must fit **all three** equations. Test each option.

$(3, 3, 0)$: first 6 correct, second $6 - 3 + 0 = 3$ correct, third $3 + 6 - 0 = 9 \neq 2$

$(4, 0, 2)$: first 6 correct, second $8 - 0 + 2 = 10 \neq 3$

$(1, 2, 3)$: first $1 + 2 + 3 = 6$, second $2 - 2 + 3 = 3$, third $1 + 4 - 3 = 2$. All three correct.

$(2, -1, -2)$: first $2 - 1 - 2 = -1 \neq 6$

Answer: $(1, 2, 3)$

Tip: Each wrong option fits two of the equations. Always check your answer in all three equations, not just one or two.

2 (3 marks)

Solve the simultaneous equations

$$p + q = 11$$

$$q + r = 4$$

$$p + r = 9$$

Label the equations (1), (2) and (3).

Add all three: $2p + 2q + 2r = 24$, so $p + q + r = 12$ (4)

$$(4) - (2): p = 12 - 4 = 8$$

$$(4) - (3): q = 12 - 9 = 3$$

$$(4) - (1): r = 12 - 11 = 1$$

(Check: $8 + 3 = 11$, $3 + 1 = 4$, $8 + 1 = 9$ all correct)

(Or: (1) – (3) gives $q - r = 2$; add (2) to get $2q = 6$, so $q = 3$, then $p = 8$ and $r = 1$.)

Answer: $p = 8$, $q = 3$, $r = 1$

Tip: Label your equations so you can say which ones you are adding or subtracting. Then check all three answers in all three equations.

Marks: 1 mark for a correct first step that removes a letter (e.g. $p + q + r = 12$ or $q - r = 2$). 1 mark for one correct value. 1 mark for all three values.

3 (3 marks)

Solve the simultaneous equations

$$x + y + z = 3$$

$$x - 2y = 2$$

$$2y + z = 0$$

The second and third equations each have only two letters, so write x and z in terms of y .

$$\text{From } x - 2y = 2: x = 2 + 2y$$

$$\text{From } 2y + z = 0: z = -2y$$

$$\text{Substitute into } x + y + z = 3: (2 + 2y) + y + (-2y) = 3$$

$$2 + y = 3, \text{ so } y = 1$$

Then $x = 2 + 2 \times 1 = 4$ and $z = -2 \times 1 = -2$

(Check: $4 + 1 - 2 = 3$, $4 - 2 = 2$, $2 - 2 = 0$ all correct)

Answer: $x = 4$, $y = 1$, $z = -2$

Tip: Watch the signs when you substitute: $z = -2y$, so $+z$ becomes $-2y$.

Marks: 1 mark for an equation in one letter, e.g. $2 + y = 3$. 1 mark for $y = 1$. 1 mark for $x = 4$ and $z = -2$.

4 (5 marks)

Solve the simultaneous equations

$$2x + 3y - z = -5$$

$$x - 2y + 3z = 22$$

$$4x + y + 2z = 20$$

Do not use trial and improvement.

You must show your working.

Label the equations (1), (2) and (3). z has coefficient -1 in (1), so eliminate z .

$$(1) \times 3: 6x + 9y - 3z = -15$$

$$\text{Add (2): } 7x + 7y = 7, \text{ so } x + y = 1 \quad (4)$$

$$(1) \times 2: 4x + 6y - 2z = -10$$

$$\text{Add (3): } 8x + 7y = 10 \quad (5)$$

$$(4) \times 7: 7x + 7y = 7$$

$$(5) - \text{this: } x = 3$$

$$\text{Substitute into (4): } 3 + y = 1, \text{ so } y = -2$$

$$\text{Substitute into (1): } 6 - 6 - z = -5, \text{ so } z = 5$$

(Check in (2): $3 + 4 + 15 = 22$ correct. In (3): $12 - 2 + 10 = 20$ correct)

Answer: $x = 3, y = -2, z = 5$

Tip: Eliminate the same letter twice, using two different pairs of equations. Write each new equation on its own line with a label; mixing up equations early on spoils everything after it.

Marks: 1 mark for eliminating one letter from one pair of equations. 1 mark for eliminating the **same** letter from a different pair. 1 mark for solving the two new equations. 1 mark for two correct values. 1 mark for all three values.

5 (5 marks)

Solve the simultaneous equations

$$4x + y + z = 3$$

$$2x - 3y + 2z = -12$$

$$6x + 2y - 3z = 15$$

Do not use trial and improvement.

You must show your working.

Label the equations (1), (2) and (3). y and z both have coefficient 1 in (1). Eliminate z .

$$(1) \times 2: 8x + 2y + 2z = 6$$

$$\text{Subtract (2): } 6x + 5y = 18 \quad (4)$$

$$(1) \times 3: 12x + 3y + 3z = 9$$

$$\text{Add (3): } 18x + 5y = 24 \quad (5)$$

$$(5) - (4): 12x = 6, \text{ so } x = \frac{1}{2}$$

$$\text{Substitute into (4): } 3 + 5y = 18, \text{ so } 5y = 15, y = 3$$

$$\text{Substitute into (1): } 2 + 3 + z = 3, \text{ so } z = -2$$

(Check in (2): $1 - 9 - 4 = -12$ correct. In (3): $3 + 6 + 6 = 15$ correct)

Answer: $x = \frac{1}{2}$, $y = 3$, $z = -2$

Tip: Not every answer is a whole number. $12x = 6$ gives $x = \frac{1}{2}$: keep going with the fraction rather than deciding you have made a mistake.

Marks: 1 mark for eliminating one letter from one pair of equations. 1 mark for eliminating the same letter from a different pair. 1 mark for solving the two new equations. 1 mark for two correct values. 1 mark for all three values.

6 (4 marks)

A curve has equation $y = ax^2 + bx + c$, where a , b and c are constants.

The curve passes through the points $(1, 4)$, $(-2, 19)$ and $(3, 14)$.

Work out the values of a , b and c .

You must show your working.

Substitute each point into $y = ax^2 + bx + c$:

$$(1, 4): a + b + c = 4 \quad (1)$$

$$(-2, 19): 4a - 2b + c = 19 \quad (2)$$

$$(3, 14): 9a + 3b + c = 14 \quad (3)$$

$$\text{Eliminate } c. (2) - (1): 3a - 3b = 15, \text{ so } a - b = 5 \quad (4)$$

$$(3) - (1): 8a + 2b = 10, \text{ so } 4a + b = 5 \quad (5)$$

$$(4) + (5): 5a = 10, \text{ so } a = 2$$

$$\text{Substitute into (4): } 2 - b = 5, \text{ so } b = -3$$

$$\text{Substitute into (1): } 2 - 3 + c = 4, \text{ so } c = 5$$

(Check: $y = 2x^2 - 3x + 5$ at $x = -2$ gives $8 + 6 + 5 = 19$, and at $x = 3$ gives $18 - 9 + 5 = 14$ correct)

Answer: $a = 2$, $b = -3$, $c = 5$

Tip: Square the x -coordinate before multiplying by a : at $x = -2$, $ax^2 = 4a$ (not $-4a$), but $bx = -2b$.

Marks: 1 mark for all three equations. 1 mark for eliminating c from two pairs. 1 mark for solving to get one value. 1 mark for all three values.

7 (5 marks)

Solve the simultaneous equations

$$2a + 3b + 4c = -1$$

$$4a - b + 2c = 9$$

$$6a + 5b - 2c = -2$$

Do not use trial and improvement.

You must show your working.

Label the equations (1), (2) and (3). The c terms in (2) and (3) are $+2c$ and $-2c$, so eliminate c .

$$(2) + (3): 10a + 4b = 7 \quad (4)$$

$$(2) \times 2: 8a - 2b + 4c = 18$$

$$(1) - \text{this}: -6a + 5b = -19 \quad (5)$$

$$(4) \times 5: 50a + 20b = 35$$

$$(5) \times 4: -24a + 20b = -76$$

$$\text{Subtract: } 74a = 111, \text{ so } a = \frac{111}{74} = \frac{3}{2}$$

$$\text{Substitute into (4): } 15 + 4b = 7, \text{ so } 4b = -8, b = -2$$

$$\text{Substitute into (2): } 6 + 2 + 2c = 9, \text{ so } 2c = 1, c = \frac{1}{2}$$

(Check in (1): $3 - 6 + 2 = -1$ correct. In (3): $9 - 10 - 1 = -2$ correct)

Answer: $a = \frac{3}{2}$, $b = -2$, $c = \frac{1}{2}$

Tip: Eliminate the **same** letter from two different pairs of equations, or you end up with two equations in three unknowns. Fractions such as $\frac{111}{74}$ should cancel: if they don't, check your arithmetic before going on.

Marks: 1 mark for eliminating one letter from one pair of equations. 1 mark for eliminating the same letter from a different pair. 1 mark for solving the two new equations. 1 mark for two correct values. 1 mark for all three values.
