

Rearranging Formulae

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Rearrange $p = 3q^3$ to make q the subject.

Circle your answer.

Options: $q = \sqrt[3]{\frac{p}{3}}$, $q = \frac{\sqrt[3]{p}}{3}$, $q = \sqrt[3]{3p}$, $q = \left(\frac{p}{3}\right)^3$

q has been cubed and then multiplied by 3.

Undo these in reverse order.

Divide both sides by 3: $\frac{p}{3} = q^3$

Cube root both sides: $q = \sqrt[3]{\frac{p}{3}}$

Answer: $q = \sqrt[3]{\frac{p}{3}}$

Tip: Divide by 3 **before** you cube root, and draw the root sign over the whole fraction.

2 (3 marks)

A flat ring has outer radius R and inner radius r .

Its area, A , is given by

$$A = \pi(R^2 - r^2)$$

Make R the subject of the formula.

Divide both sides by π : $\frac{A}{\pi} = R^2 - r^2$

Add r^2 to both sides: $R^2 = \frac{A}{\pi} + r^2$

Square root both sides: $R = \sqrt{\frac{A}{\pi} + r^2}$

Answer: $R = \sqrt{\frac{A}{\pi} + r^2}$ (or $\sqrt{\frac{A + \pi r^2}{\pi}}$)

Tip: The square root must cover the whole right-hand side. $\sqrt{\frac{A}{\pi} + r^2}$ is **not** $\sqrt{\frac{A}{\pi}} + r$.

Marks: 1 mark for $\frac{A}{\pi} = R^2 - r^2$ (or $A = \pi R^2 - \pi r^2$). 1 mark for $R^2 = \frac{A}{\pi} + r^2$ or equivalent. 1 mark for the correct answer.

3 (3 marks)

Make a the subject of $3(a - 2b) = ab + 5$

Expand the bracket: $3a - 6b = ab + 5$

a appears twice, so collect the a terms on the left and everything else on the right:

$$3a - ab = 5 + 6b$$

Factorise the left side (take out a): $a(3 - b) = 5 + 6b$

Divide both sides by $(3 - b)$.

Answer: $a = \frac{6b + 5}{3 - b}$ (or $\frac{-6b - 5}{b - 3}$)

Tip: An answer with a on both sides is not finished. Collect the a terms together and factorise.

Marks: 1 mark for collecting the a terms on one side, e.g. $3a - ab = 5 + 6b$. 1 mark for $a(3 - b)$. 1 mark for the correct answer.

4 (3 marks)

Make y the subject of $\frac{2}{x} = \frac{1}{y} - \frac{3}{z}$

Get the y term on its own: add $\frac{3}{z}$ to both sides.

$$\frac{1}{y} = \frac{2}{x} + \frac{3}{z}$$

Write the right side as a single fraction, with common denominator xz :

$$\frac{1}{y} = \frac{2z}{xz} + \frac{3x}{xz} = \frac{2z + 3x}{xz}$$

There is now one fraction on each side, so turn both sides upside down:

$$y = \frac{xz}{2z + 3x}$$

Answer: $y = \frac{xz}{2z + 3x}$

Tip: You can only turn both sides upside down when each side is a **single** fraction. Flipping each term separately to get $y = \frac{x}{2} + \frac{z}{3}$ is wrong.

Marks: 1 mark for $\frac{1}{y} = \frac{2}{x} + \frac{3}{z}$. 1 mark for $\frac{2z + 3x}{xz}$. 1 mark for the correct answer.

5 (4 marks)

Rearrange $y = \sqrt{\frac{x+4}{x-1}}$ to make x the subject.

Square both sides to remove the square root: $y^2 = \frac{x+4}{x-1}$

Multiply both sides by $(x-1)$: $y^2(x-1) = x+4$

Expand: $xy^2 - y^2 = x + 4$

Collect the x terms on the left: $xy^2 - x = y^2 + 4$

Factorise: $x(y^2 - 1) = y^2 + 4$

Divide both sides by $(y^2 - 1)$.

Answer: $x = \frac{y^2 + 4}{y^2 - 1}$

Tip: Use a bracket when you multiply: $y^2(x-1) = xy^2 - y^2$. Writing $xy^2 - 1$ loses a term and makes the rest impossible.

Marks: 1 mark for squaring both sides. 1 mark for $y^2(x-1) = x+4$. 1 mark for collecting the x terms and factorising. 1 mark for the correct answer.

6 (4 marks)

Rearrange $m = \frac{4 - n^3}{2n^3} + 1$ to make n the subject.

Subtract 1 from both sides: $m - 1 = \frac{4 - n^3}{2n^3}$

Multiply both sides by $2n^3$: $2n^3(m - 1) = 4 - n^3$

Expand: $2mn^3 - 2n^3 = 4 - n^3$

Collect the n^3 terms on the left: $2mn^3 - 2n^3 + n^3 = 4$, so $2mn^3 - n^3 = 4$

Factorise: $n^3(2m - 1) = 4$

Divide by $(2m - 1)$: $n^3 = \frac{4}{2m - 1}$

Cube root both sides.

Answer: $n = \sqrt[3]{\frac{4}{2m - 1}}$

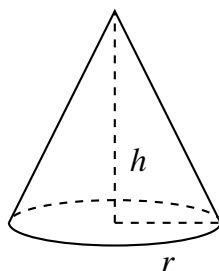
Tip: Treat n^3 like a single letter until the end. Don't forget the final cube root, and make the root sign cover the whole fraction.

Marks: 1 mark for clearing the fraction correctly. 1 mark for collecting the n^3 terms on one side.
1 mark for $n^3 = \frac{4}{2m - 1}$. 1 mark for the correct answer.

7 (5 marks)

A cone has base radius r and perpendicular height h .
The curved surface area, S , of the cone is given by

$$S = \pi r \sqrt{r^2 + h^2}$$



Not drawn accurately

- (a) Make h the subject of the formula.

Divide both sides by πr : $\frac{S}{\pi r} = \sqrt{r^2 + h^2}$

Square both sides: $\frac{S^2}{\pi^2 r^2} = r^2 + h^2$

Subtract r^2 : $h^2 = \frac{S^2}{\pi^2 r^2} - r^2$

Square root both sides.

Answer: $h = \sqrt{\frac{S^2}{\pi^2 r^2} - r^2}$

Tip: Square **everything** on the left: $\left(\frac{S}{\pi r}\right)^2 = \frac{S^2}{\pi^2 r^2}$. And $\sqrt{r^2 + h^2}$ is not $r + h$, so you must square before you can get at h .

Marks: 1 mark for dividing by πr and squaring both sides. 1 mark for $h^2 = \frac{S^2}{\pi^2 r^2} - r^2$. 1 mark for the correct answer.

- (b) A cone has base radius 4 cm and curved surface area 150 cm^2 .
Work out the perpendicular height of the cone.
Give your answer to 3 significant figures.

Substitute $S = 150$ and $r = 4$ into the formula from part (a):

$$h = \sqrt{\frac{150^2}{\pi^2 \times 4^2} - 4^2}$$

$$\frac{150^2}{16\pi^2} = 142.49 \dots$$

$$h = \sqrt{142.49 \dots - 16} = \sqrt{126.49 \dots} = 11.246 \dots$$

Answer: 11.2 cm

Tip: Keep the full calculator value until the end; rounding 142.49 ... early can change the final answer. Put brackets round $\pi^2 \times 4^2$ on your calculator.

Marks: 1 mark for substituting correctly into a correct formula. 1 mark for 11.2.

8 (3 marks)

Solve the simultaneous equations

$$ax + 2y = 7$$

$$3x - y = a$$

where a is a constant.

Give x and y in terms of a .

Rearrange the second equation: $y = 3x - a$

Substitute into the first: $ax + 2(3x - a) = 7$

$$ax + 6x - 2a = 7$$

Collect the x terms and factorise: $x(a + 6) = 7 + 2a$

$$x = \frac{2a + 7}{a + 6}$$

$$\text{Now find } y: y = 3x - a = \frac{3(2a + 7)}{a + 6} - \frac{a(a + 6)}{a + 6}$$

$$y = \frac{6a + 21 - a^2 - 6a}{a + 6} = \frac{21 - a^2}{a + 6}$$

Answer: $x = \frac{2a + 7}{a + 6}, \quad y = \frac{21 - a^2}{a + 6}$

Tip: Solve for x and y , not a : a is a constant, so treat it like a number. Collect the x terms and factorise, just as when x appears twice in a formula.

Marks: 1 mark for eliminating y (or x) correctly, e.g. $ax + 6x - 2a = 7$. 1 mark for $x = \frac{2a + 7}{a + 6}$. 1 mark for y correct.
