

Product Rule for Counting

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

A suitcase lock has three wheels.

Each wheel can be set to any digit from 0 to 9

How many different codes can be set on the lock?

Circle your answer.

Options: 30, 720, 999, 1000

Each wheel has 10 choices (0, 1, 2, ..., 9), and digits may repeat.

Product rule: $10 \times 10 \times 10 = 1000$

Answer: 1000

Tip: 000 is a code too, so the answer is not 999. Digits can repeat on a lock, so it is not $10 \times 9 \times 8 = 720$.

2 (2 marks)

Nine runners take part in a race.

There are no ties.

In how many different ways can the first, second and third places be filled?

First place: any of the 9 runners, so 9 choices.

Second place: any runner except the winner, so 8 choices.

Third place: 7 choices.

Product rule: $9 \times 8 \times 7 = 504$

Answer: 504

Tip: The order matters here (first is different from second), so do not divide by anything. Multiply the number of choices for each place in turn.

Marks: 1 mark for $9 \times 8 \times 7$. 1 mark for 504.

3 (3 marks)

Four-digit integers are made using the digits

1 2 4 5 7 9

No digit is used more than once in an integer.

(a) How many different integers can be made?

No digit can be repeated, so the number of choices drops by 1 each time.

$$6 \times 5 \times 4 \times 3 = 360$$

Answer: 360

(b) How many of the integers are greater than 5000 **and** even?

Fill the restricted places first.

First digit: greater than 5000 means it starts with 5, 7 or 9, so 3 choices.

Last digit: even means it ends in 2 or 4, so 2 choices.

These two sets of digits do not overlap, so the choices do not affect each other.

Second digit: 4 digits are left, so 4 choices.

Third digit: 3 choices.

$$3 \times 4 \times 3 \times 2 = 72$$

Answer: 72

Tip: Deal with the restricted positions (first and last digit) **before** the others. Starting from the left with 6, 5, 4, ... goes wrong.

Marks: 1 mark for 3 choices for the first digit and 2 for the last (or 3×2 seen). 1 mark for 72.

4 (2 marks)

How many four-digit integers have four different digits?

First digit: a four-digit integer cannot start with 0, so 1 to 9 gives 9 choices.

Second digit: 0 is now allowed, but not the digit already used, so 9 choices.

Third digit: 8 choices.

Fourth digit: 7 choices.

$$9 \times 9 \times 8 \times 7 = 4536$$

Answer: 4536

Tip: Remember that 0 is a digit. It cannot go first, but it can go in any other place, so the second digit has 9 choices, not 8.

Marks: 1 mark for $9 \times 9 \times 8 \times 7$. 1 mark for 4536.

5 (4 marks)

Six friends, including Priya and Tom, sit in a row of six seats at the cinema.

- (a) Priya and Tom sit in the two end seats.
In how many different ways can the six friends sit?

Priya and Tom can sit at the ends in 2 ways (Priya on the left or Tom on the left).

The other 4 friends fill the 4 middle seats: $4 \times 3 \times 2 \times 1 = 24$ ways.

$$2 \times 24 = 48$$

Answer: 48

Tip: Don't forget that Priya and Tom can swap ends. That doubles the 24.

Marks: 1 mark for $2 \times 4 \times 3 \times 2 \times 1$ or for 24. 1 mark for 48.

- (b) Priya and Tom sit next to each other.
In how many different ways can the six friends sit?

Count the places for the pair of seats Priya and Tom use.

Seats 1-2, 2-3, 3-4, 4-5 or 5-6, so 5 choices.

Priya and Tom can sit in either order, so 2 choices.

The other 4 friends fill the 4 remaining seats: $4 \times 3 \times 2 \times 1 = 24$

$$5 \times 2 \times 24 = 240$$

Answer: 240

Tip: There are 5 pairs of seats next to each other in a row of 6, not 6.

Marks: 1 mark for two of 5, 2 and 24 (or $4 \times 3 \times 2 \times 1$) in a product. 1 mark for 240.

6 (3 marks)

A netball club's kit is one shirt and one pair of shorts.

The shirts come in n colours.

The shorts come in $(n - 4)$ colours.

There are 96 different kits possible.

Work out the value of n .

You must show your working.

Product rule: number of kits = $n \times (n - 4)$

$$\text{So } n(n - 4) = 96$$

$$\text{Expand and rearrange: } n^2 - 4n - 96 = 0$$

$$\text{Factorise: } (n - 12)(n + 8) = 0$$

$$n = 12 \text{ or } n = -8$$

n is a number of colours, so it must be positive: $n = 12$

Check: 12 shirts and 8 pairs of shorts give $12 \times 8 = 96$ ✓

Answer: $n = 12$

Tip: Use the product rule to write an equation, then solve the quadratic. Reject $n = -8$: you cannot have a negative number of colours.

Marks: 1 mark for $n(n - 4) = 96$. 1 mark for $n^2 - 4n - 96 = 0$ factorised or solved. 1 mark for $n = 12$ only.

7 (3 marks)

Five-digit integers are made using each of these digits exactly once.

0 3 4 7 8

How many of the integers are even?

You must show your working.

An even integer must end in 0, 4 or 8. The first digit cannot be 0.

Split into cases by the last digit.

Case 1: last digit 0. The other four digits (3, 4, 7, 8) can go in any order:

$$4 \times 3 \times 2 \times 1 = 24$$

Case 2: last digit 4. First digit: 3, 7 or 8 (not 0, not 4), so 3 choices.

Then 3, 2 and 1 choices for the middle three digits: $3 \times 3 \times 2 \times 1 = 18$

Case 3: last digit 8. In the same way: $3 \times 3 \times 2 \times 1 = 18$

Add the cases: $24 + 18 + 18 = 60$

Answer: 60

Tip: The 0 makes this harder. When 0 is at the end, the first digit has 4 choices; when it isn't, the first digit has only 3 choices. So split into cases and **add** them.

Marks: 1 mark for 24 (last digit 0). 1 mark for 18 (last digit 4 or 8). 1 mark for 60.

8 (3 marks)

How many odd integers between 3000 and 7000 have four different digits?
You must show your working.

The first digit is 3, 4, 5 or 6. The last digit is 1, 3, 5, 7 or 9.

Two of the possible first digits (3 and 5) are odd, so split into cases.

First digit odd (3 or 5): the last digit is odd but not the first digit, so 4 choices.

The middle two digits: 8 choices, then 7 choices.

$$2 \times 4 \times 8 \times 7 = 448$$

First digit even (4 or 6): the last digit can be any of the 5 odd digits.

$$2 \times 5 \times 8 \times 7 = 560$$

$$\text{Total: } 448 + 560 = 1008$$

Answer: 1008

Tip: $4 \times 5 \times 8 \times 7 = 1120$ is wrong: it counts integers such as 3483, where the first and last digits are the same. Split into cases by the first digit, then add.

Marks: 1 mark for splitting into cases (or one of 448 and 560). 1 mark for both 448 and 560. 1 mark for 1008.
