

Piecewise Functions

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A function f is given by

$$\begin{aligned} f(x) &= x^2 + 1 & x < 2 \\ &= 9 - 2x & x \geq 2 \end{aligned}$$

(a) Work out $f(-3)$

$-3 < 2$, so use the first piece, $x^2 + 1$.

$$f(-3) = (-3)^2 + 1 = 9 + 1 = 10$$

Answer: 10

Tip: Check which domain the input is in before you substitute. $(-3)^2 = 9$, not -9 .

(b) Work out $f(4)$

$4 \geq 2$, so use the second piece, $9 - 2x$.

$$f(4) = 9 - 2 \times 4 = 1$$

Answer: 1

2 (1 mark)

A function f is given by

$$f(x) = x + 3 \quad -3 \leq x < 0$$

$$= 3 - x^2 \quad 0 \leq x \leq 2$$

Circle the range of f .

Options: $0 \leq f(x) \leq 3$, $-1 \leq f(x) \leq 3$, $-3 \leq f(x) \leq 2$, $-1 \leq f(x) < 3$

First piece: from $f(-3) = 0$ up towards 3 (but $x = 0$ is not in this piece).

Second piece: $f(0) = 3$ down to $f(2) = 3 - 4 = -1$

So the outputs go from -1 up to 3, and 3 is reached (at $x = 0$).

Range: $-1 \leq f(x) \leq 3$

Answer: $-1 \leq f(x) \leq 3$

Tip: Look at the outputs of every piece. The value 3 is missed by the first piece but reached by the second, so the range includes it.

3 (3 marks)

A function f is given by

$$f(x) = 2x + 1 \quad 0 \leq x < 2$$

$$= 5 \quad 2 \leq x < 5$$

$$= 15 - 2x \quad 5 \leq x \leq 7$$

On the grid, draw the graph of $y = f(x)$ for $0 \leq x \leq 7$

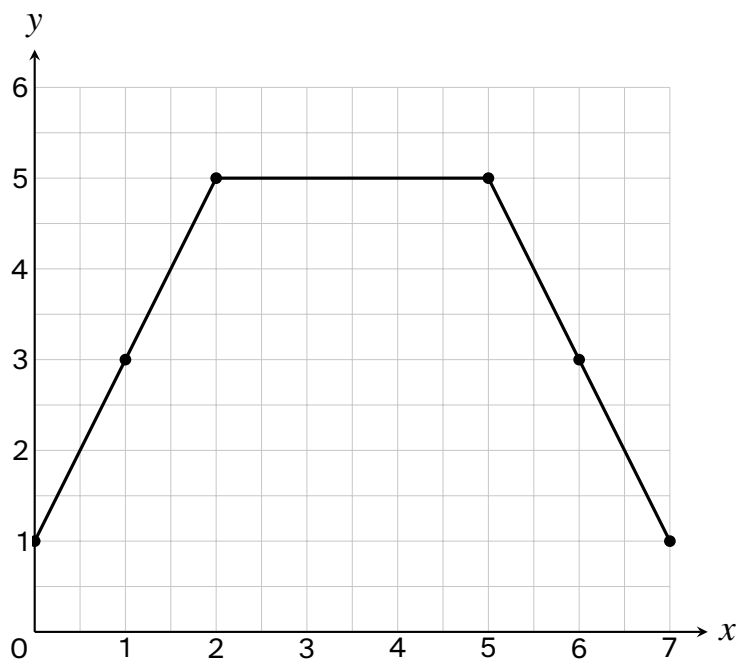
Work out the value at the start and end of each piece.

$2x + 1$: $x = 0$ gives 1, $x = 2$ gives 5.

5: a horizontal line at $y = 5$ from $x = 2$ to $x = 5$.

$15 - 2x$: $x = 5$ gives 5, $x = 7$ gives 1.

The pieces join up. Draw each straight piece with a ruler.



Tip: Use a ruler for straight-line pieces; freehand lines lose marks. Each piece must start and stop at the right x values.

Marks: 1 mark for each correct piece.

4 (5 marks)

A function f is given by

$$\begin{aligned} f(x) &= x^2 - 2x + 2 & 0 \leq x < 3 \\ &= 8 - x & 3 \leq x < 5 \\ &= 2x - 7 & 5 \leq x \leq 6 \end{aligned}$$

(a) On the grid, draw the graph of $y = f(x)$

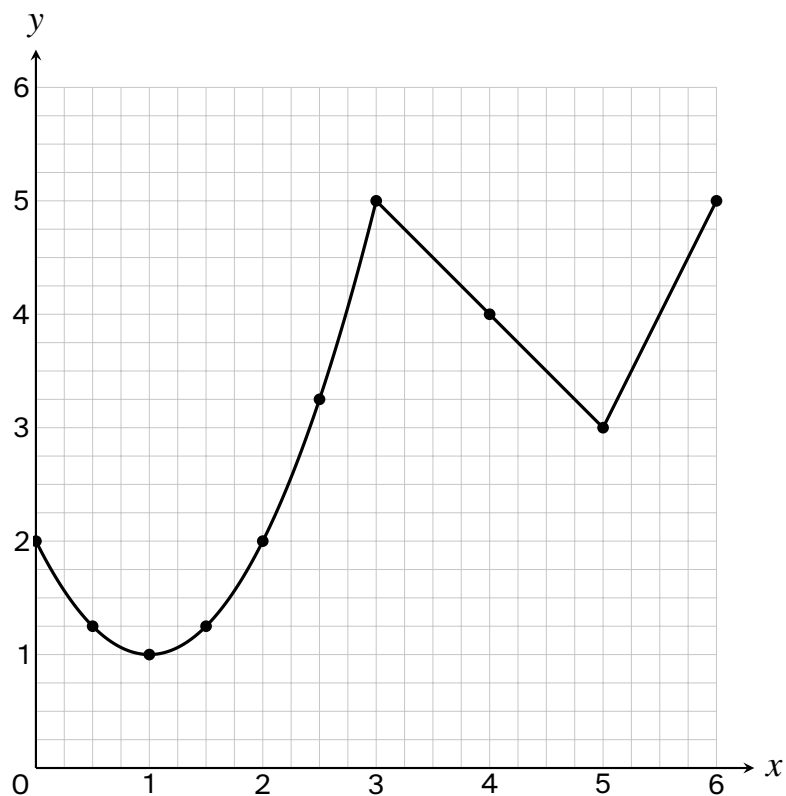
First piece: work out some points of $x^2 - 2x + 2$:

x	0	0.5	1	1.5	2	2.5	3
y	2	1.25	1	1.25	2	3.25	5

Plot them and join with a smooth curve (minimum at $(1, 1)$).

Second piece: $8 - x$ goes from $(3, 5)$ to $(5, 3)$. Use a ruler.

Third piece: $2x - 7$ goes from $(5, 3)$ to $(6, 5)$. Use a ruler.



Tip: The curved piece must be a smooth curve through the plotted points, not straight lines joining them. Plot several points, including the minimum.

Marks: 1 mark for each straight piece. 2 marks for the curve (1 mark for at least 3 correct points).

- (b) How many solutions does the equation $f(x) = 4$ have?

Draw the line $y = 4$ across your graph and count the crossings.

$x = 1 + \sqrt{3} \approx 2.7$ (curve: $x^2 - 2x + 2 = 4$; the other root, $1 - \sqrt{3} \approx -0.7$, is not in $0 \leq x < 3$).

$x = 4$ (second piece: $8 - x = 4$).

$x = 5.5$ (third piece: $2x - 7 = 4$).

So there are 3 solutions.

Answer: 3

Tip: A solution only counts if it is inside the domain of the piece it came from.

5 (2 marks)

A function f is given by

$$\begin{aligned} f(x) &= 2x + c & x < 3 \\ &= x^2 - 4 & x \geq 3 \end{aligned}$$

c is a constant.

The two parts of the graph of $y = f(x)$ join where $x = 3$

Work out the value of c .

The pieces join at $x = 3$, so both expressions give the same value there.

$$2 \times 3 + c = 3^2 - 4$$

$$6 + c = 5$$

$$c = -1$$

Answer: $c = -1$

Tip: Substitute the joining value $x = 3$ into **both** expressions and set them equal. You don't need inequalities.

Marks: 1 mark for $6 + c = 5$. 1 mark for $c = -1$.

6 (3 marks)

A function f is given by

$$\begin{aligned} f(x) &= 10 - 3x & x < 2 \\ &= x^2 & x \geq 2 \end{aligned}$$

Solve $f(x) = 7$

Solve with each piece, then check the domain.

First piece: $10 - 3x = 7$, so $3x = 3$, $x = 1$. $1 < 2$, so this counts.

Second piece: $x^2 = 7$, so $x = \sqrt{7} \approx 2.65$ or $x = -\sqrt{7} \approx -2.65$

$\sqrt{7} \geq 2$, so it counts. $-\sqrt{7}$ is not ≥ 2 , so reject it.

Solutions: $x = 1$ and $x = \sqrt{7}$

Answer: $x = 1$ and $x = \sqrt{7}$ (= 2.65 to 3 s.f.)

Tip: After solving, test each answer against the domain of the piece it came from. $x = -\sqrt{7}$ solves $x^2 = 7$, but the second piece is only used for $x \geq 2$.

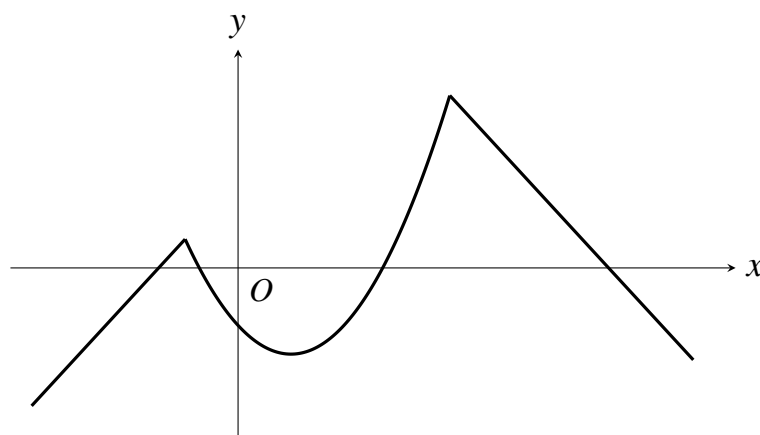
Marks: 1 mark for $x = 1$. 1 mark for $x = \pm\sqrt{7}$. 1 mark for rejecting $-\sqrt{7}$.

7 (4 marks)

A function f is given by

$$\begin{aligned} f(x) &= 2x + 3 & x < -1 \\ &= x^2 - 2x - 2 & -1 \leq x \leq 4 \\ &= 14 - 2x & x > 4 \end{aligned}$$

A sketch of $y = f(x)$ is shown.



Not drawn accurately

Solve $f(x) = 2$

You must show your working.

Solve with each piece and keep only answers in that piece's domain.

First piece: $2x + 3 = 2$, $x = -\frac{1}{2}$. But $-\frac{1}{2}$ is not < -1 , so reject it.

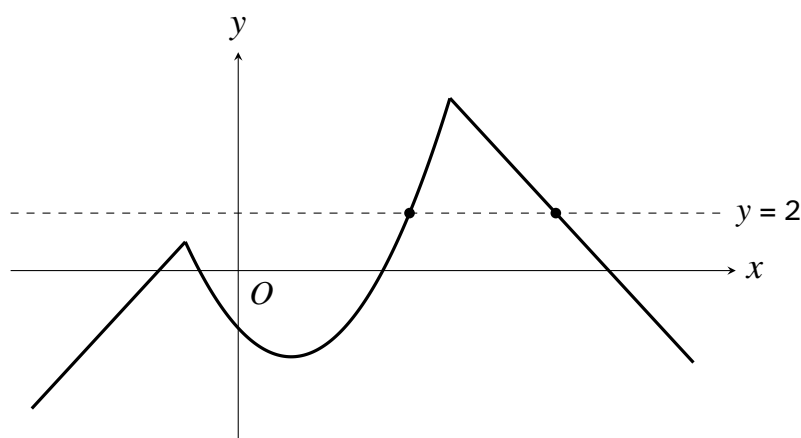
Second piece: $x^2 - 2x - 2 = 2$, so $x^2 - 2x - 4 = 0$

Quadratic formula: $x = \frac{2 \pm \sqrt{4 + 16}}{2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

$x = 1 + \sqrt{5} \approx 3.24$ is in $-1 \leq x \leq 4$ ✓ $x = 1 - \sqrt{5} \approx -1.24$ is less than -1 , so reject it.

Third piece: $14 - 2x = 2$, $x = 6$. $6 > 4$ ✓

The sketch agrees: the line $y = 2$ crosses the graph twice.



Answer: $x = 1 + \sqrt{5}$ ($= 3.24$ to 3 s.f.) and $x = 6$

Tip: Use the sketch to see how many solutions to expect. Here $x = -\frac{1}{2}$ and $x = 1 - \sqrt{5}$ each solve an equation, but they are outside the domain of the piece they came from, so they are not solutions.

Marks: 1 mark for $x^2 - 2x - 4 = 0$. 1 mark for $x = 1 \pm \sqrt{5}$. 1 mark for $x = 6$. 1 mark for rejecting $x = -\frac{1}{2}$ and $x = 1 - \sqrt{5}$ (no extra answers).

8 (4 marks)

A function f is given by

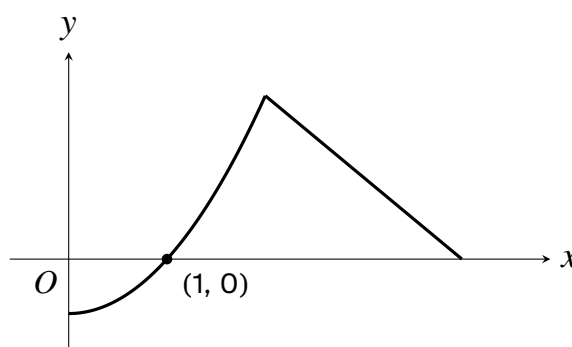
$$\begin{aligned} f(x) &= px^2 + q & 0 \leq x < 2 \\ &= 12 - 3x & 2 \leq x \leq 4 \end{aligned}$$

p and q are constants.

The two parts of the graph join where $x = 2$

The graph passes through the point $(1, 0)$

A sketch of $y = f(x)$ is shown.



Not drawn accurately

Work out the values of p and q .

Find the point where the pieces join, using the linear piece at $x = 2$:

$f(2) = 12 - 3 \times 2 = 6$, so the join is at $(2, 6)$.

$(2, 6)$ is on $y = px^2 + q$: $4p + q = 6$

$(1, 0)$ is on $y = px^2 + q$: $p + q = 0$

Subtract the second equation from the first: $3p = 6$, so $p = 2$

Substitute: $2 + q = 0$, so $q = -2$

Check: $2 \times 2^2 - 2 = 6$ ✓ and $2 \times 1^2 - 2 = 0$ ✓

Answer: $p = 2$, $q = -2$

Tip: Use the linear piece to find the coordinates of the joining point. Then you have two points on the curve, which give two simultaneous equations.

Marks: 1 mark for the join at $(2, 6)$. 1 mark for both equations. 1 mark for one correct value. 1 mark for both values.