

Maximum and Minimum Problems

For AQA Level 2 Certificate in Further Mathematics (8365)

Name _____

Total marks: **26**

Answer **all** questions in the spaces provided. Show your working: marks are given for correct method. You may use a calculator.

- 1** A ball is thrown upwards from a balcony.
After t seconds, the height of the ball above the ground is h metres, where

$$h = 2 + 12t - 5t^2$$

Use calculus to work out the maximum height of the ball.

[3 marks]

Answer _____ m

2 $y = 5x + \frac{80}{x}$, where $x > 0$

- (a) Work out the value of x for which $\frac{dy}{dx} = 0$

[2 marks]

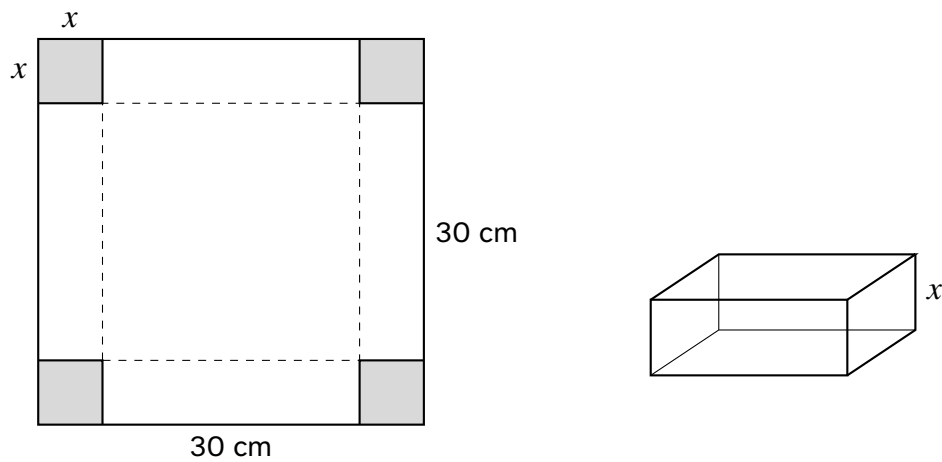
Answer $x =$ _____

- (b) Use $\frac{d^2y}{dx^2}$ to show that y has a minimum value at this value of x .
Work out the minimum value of y .

[3 marks]

Answer _____

- 3** A square piece of card has sides of length 30 cm.
 A square of side x cm is cut from each corner.
 The card is then folded along the dashed lines to make an open box with no lid.



Not drawn accurately

- (a) Show that the volume of the box, $V \text{ cm}^3$, is given by

$$V = 4x^3 - 120x^2 + 900x$$

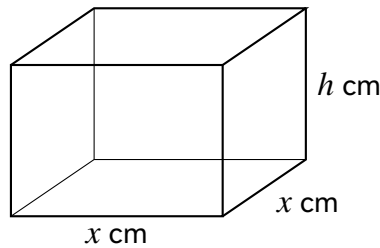
[2 marks]

- (b) Use calculus to work out the maximum volume of the box.

[4 marks]

Answer _____ cm^3

- 4 A metal water tank is an open cuboid with no top.
The base is a square of side x cm. The height is h cm.
The volume of the tank is 4000 cm^3 .
The total area of metal used for the tank is $A \text{ cm}^2$.



Not drawn accurately

- (a) Show that $A = x^2 + \frac{16000}{x}$

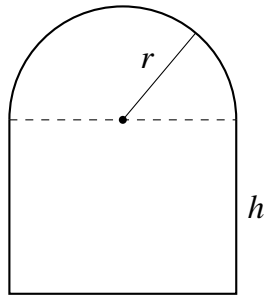
[2 marks]

- (b) Use calculus to work out the minimum value of A .
You must show that your value is a minimum.

[4 marks]

Answer _____ cm^2

- 5 A window is in the shape of a rectangle with a semicircle on top. The semicircle has radius r metres. The rectangle has height h metres. The frame around the outside edge of the window has a total length of 10 m. The area of the window is A m².



Not drawn accurately

- (a) Show that $A = 10r - 2r^2 - \frac{1}{2}\pi r^2$

[3 marks]

- (b) Use calculus to work out the maximum area of the window. Give your answer to 3 significant figures.

[3 marks]

Answer _____ m²