

Linear and Quadratic Equations in Context

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the solutions of $(x - 4)^2 = 7$

Options: $x = 4 + \sqrt{7}$ only, $x = 4 \pm \sqrt{7}$, $x = -4 \pm \sqrt{7}$, $x = 11$ and $x = -3$

Don't expand the bracket. Square root both sides instead.

$x - 4 = \pm\sqrt{7}$ (a square root can be positive or negative)

Add 4 to both sides: $x = 4 \pm \sqrt{7}$

Answer: $x = 4 \pm \sqrt{7}$

Tip: Remember the $\pm$: there are two solutions, $4 + \sqrt{7}$ and $4 - \sqrt{7}$. Forgetting the second one is the most common mistake here.

2 (2 marks)

Solve $3x^2 - 4x - 9 = 0$

Give your answers to 3 significant figures.

It doesn't factorise, so use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 3$, $b = -4$, $c = -9$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times (-9)}}{2 \times 3}$$

$$b^2 - 4ac = 16 + 108 = 124$$

$$x = \frac{4 \pm \sqrt{124}}{6}$$

$$x = \frac{4 + \sqrt{124}}{6} = 2.5225 \dots \text{ or } x = \frac{4 - \sqrt{124}}{6} = -1.1892 \dots$$

Answer: $x = 2.52$ and $x = -1.19$

Tip: Write the substitution out in full, with negative numbers in brackets. Then give **both** answers to 3 significant figures: -1.19 , not -1.2 .

Marks: 1 mark for substituting correctly into the formula. 1 mark for both answers to 3 significant figures.

3 (3 marks)

Solve $\frac{5k + 3}{2k - 1} = \frac{9}{4}$

Multiply both sides by 4 and by $(2k - 1)$ (cross-multiply):

$$4(5k + 3) = 9(2k - 1)$$

Expand: $20k + 12 = 18k - 9$

Subtract $18k$ from both sides: $2k + 12 = -9$

Subtract 12 from both sides: $2k = -21$

Divide by 2: $k = -10.5$

Answer: $k = -10.5$

Tip: Cross-multiply to make **one** equation. Don't write $5k + 3 = 9$ and $2k - 1 = 4$: that gives two different answers, and neither is right.

Marks: 1 mark for $4(5k + 3) = 9(2k - 1)$. 1 mark for expanding and collecting terms correctly. 1 mark for $k = -10.5$.

4 (3 marks)

Before a sale, a coat cost $\pounds(4x + 40)$.

In the sale, the price of the coat is reduced by 20%.

At the same time, the price of a pair of boots is increased by 25%.

Before the increase, the boots cost $\pounds(2x + 48)$.

After these changes, the coat and the boots cost the same.

Work out the price of the coat in the sale.

Reducing by 20% means multiplying by 0.8. Increasing by 25% means multiplying by 1.25.

The new prices are equal: $0.8(4x + 40) = 1.25(2x + 48)$

Expand: $3.2x + 32 = 2.5x + 60$

Subtract $2.5x$ and 32 from both sides: $0.7x = 28$

Divide by 0.7: $x = 40$

Price of the coat in the sale = $0.8 \times (4 \times 40 + 40) = 0.8 \times 200 = 160$

(Check: the boots now cost $1.25 \times (2 \times 40 + 48) = 1.25 \times 128 = 160$)

Answer: $\pounds 160$

Tip: Use multipliers and keep the brackets: $0.8(4x + 40)$, not $4x + 40 - 20$. And answer the question: it asks for the sale price, not x .

Marks: 1 mark for $0.8(4x + 40) = 1.25(2x + 48)$. 1 mark for $x = 40$. 1 mark for $\pounds 160$.

5 (3 marks)

A length of rope is cut into two pieces.

The pieces have lengths $(2x + 3)$ metres and $(x + 6)$ metres.

The ratio of the lengths is $(2x + 3) : (x + 6) = 5 : 3$

Work out the length of the rope before it was cut.

If $a : b = 5 : 3$ then $3a = 5b$.

So $3(2x + 3) = 5(x + 6)$

Expand: $6x + 9 = 5x + 30$

So $x = 21$

The pieces are $2 \times 21 + 3 = 45$ m and $21 + 6 = 27$ m.

(Check: $45:27 = 5:3$, dividing both by 9)

Total length = $45 + 27 = 72$ m

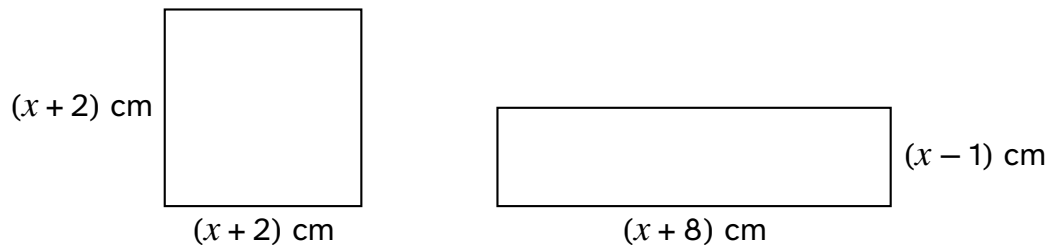
Answer: 72 metres

Tip: Turn the ratio into an equation by cross-multiplying: $\frac{2x+3}{x+6} = \frac{5}{3}$ gives $3(2x+3) = 5(x+6)$.
Writing $2x+3 = 5$ and $x+6 = 3$ does not work.

Marks: 1 mark for $3(2x+3) = 5(x+6)$. 1 mark for $x = 21$. 1 mark for 72.

6 (3 marks)

The diagram shows a square and a rectangle.
The square and the rectangle have the same area.



Not drawn accurately

Work out the perimeter of the rectangle.

$$\text{Area of the square} = (x + 2)^2 = x^2 + 4x + 4$$

$$\text{Area of the rectangle} = (x + 8)(x - 1) = x^2 + 7x - 8$$

$$\text{The areas are equal: } x^2 + 4x + 4 = x^2 + 7x - 8$$

$$\text{The } x^2 \text{ terms cancel: } 4x + 4 = 7x - 8$$

$$\text{So } 12 = 3x, \quad x = 4$$

The rectangle is $4 + 8 = 12$ cm by $4 - 1 = 3$ cm.

(Check: both areas are 36 cm^2)

$$\text{Perimeter} = 2 \times (12 + 3) = 30 \text{ cm}$$

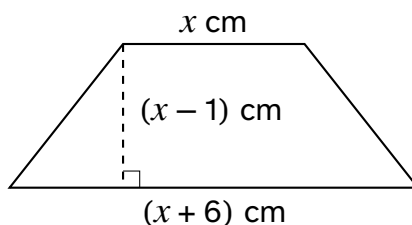
Answer: 30 cm

Tip: $(x + 2)^2$ is $x^2 + 4x + 4$, not $x^2 + 4$. Then find the sides of the rectangle and its perimeter:
 $x = 4$ is not the answer.

Marks: 1 mark for $(x + 2)^2 = (x + 8)(x - 1)$. 1 mark for $x = 4$. 1 mark for 30.

7 (4 marks)

The diagram shows a trapezium.
 The area of the trapezium is 30 cm^2 .



Not drawn accurately

- (a) Show that $x^2 + 2x - 33 = 0$

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

$$\frac{1}{2}(x + (x + 6))(x - 1) = 30$$

$$\frac{1}{2}(2x + 6) = x + 3, \text{ so } (x + 3)(x - 1) = 30$$

$$\text{Expand: } x^2 - x + 3x - 3 = 30$$

$$x^2 + 2x - 3 = 30$$

$$\text{Subtract 30: } x^2 + 2x - 33 = 0 \text{ as required}$$

Answer: Shown: $(x + 3)(x - 1) = 30$ gives $x^2 + 2x - 33 = 0$

Tip: In a “show that” every step must be written down. Show the expansion of $(x + 3)(x - 1)$; don’t jump straight to the answer.

Marks: 1 mark for $\frac{1}{2}(x + x + 6)(x - 1) = 30$. 1 mark for expanding and rearranging to the given equation.

- (b) Work out the value of x .
Give your answer to 3 significant figures.

Use the formula with $a = 1$, $b = 2$, $c = -33$:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-33)}}{2} = \frac{-2 \pm \sqrt{136}}{2}$$

$$x = 4.8309 \dots \text{ or } x = -6.8309 \dots$$

$x - 1$ is a length, so x must be more than 1. Reject -6.83 .

So $x = 4.83$

Answer: $x = 4.83$

Tip: The formula gives two values, but a side of $(x - 1)$ cm must be positive. Give the one valid answer and say why you rejected the other.

Marks: 1 mark for substituting correctly into the formula. 1 mark for 4.83 with the negative value rejected.

8 (3 marks)

The n th term of a sequence is $\frac{n^2 + 21}{n + 3}$

One term of the sequence is equal to 7.

Work out which term it is.

You must show your working.

Put the n th term equal to 7: $\frac{n^2 + 21}{n + 3} = 7$

Multiply both sides by $(n + 3)$: $n^2 + 21 = 7n + 21$

Rearrange so one side is 0: $n^2 - 7n = 0$

Factorise: $n(n - 7) = 0$

So $n = 0$ or $n = 7$

n is the position of a term, so it must be a positive whole number. Reject $n = 0$.

(Check: $\frac{7^2 + 21}{7 + 3} = \frac{70}{10} = 7$)

Answer: The 7th term

Tip: Don't divide both sides by n : you would lose a solution without showing why it doesn't count. Factorise, then explain why $n = 0$ is not a term.

Marks: 1 mark for $n^2 + 21 = 7(n + 3)$. 1 mark for $n(n - 7) = 0$ or $n = 7$. 1 mark for the 7th term with $n = 0$ rejected.

9 (4 marks)

Solve $x(x + 4)^2 = 2x(x + 9)$

Give the non-zero solutions in the form $a \pm \sqrt{b}$, where a and b are integers.

Do not use trial and improvement.

You must show your working.

Expand: $x(x + 4)^2 = x(x^2 + 8x + 16) = x^3 + 8x^2 + 16x$

and $2x(x + 9) = 2x^2 + 18x$

So $x^3 + 8x^2 + 16x = 2x^2 + 18x$

Rearrange so one side is 0: $x^3 + 6x^2 - 2x = 0$

Take out the common factor x : $x(x^2 + 6x - 2) = 0$

So $x = 0$ or $x^2 + 6x - 2 = 0$

Complete the square: $(x + 3)^2 - 9 - 2 = 0$

$(x + 3)^2 = 11$

$x + 3 = \pm\sqrt{11}$, so $x = -3 \pm \sqrt{11}$

(The formula gives the same: $x = \frac{-6 \pm \sqrt{36 + 8}}{2} = \frac{-6 \pm 2\sqrt{11}}{2} = -3 \pm \sqrt{11}$)

Answer: $x = 0$, $x = -3 + \sqrt{11}$, $x = -3 - \sqrt{11}$

Tip: Never divide both sides by x ; take x out as a factor, so $x = 0$ is one solution. Then simplify the surd: $\frac{-6 \pm \sqrt{44}}{2}$ is not yet in the form $a \pm \sqrt{b}$.

Marks: 1 mark for expanding to $x^3 + 8x^2 + 16x$. 1 mark for $x(x^2 + 6x - 2) = 0$. 1 mark for completing the square or substituting into the formula. 1 mark for $-3 \pm \sqrt{11}$.
