

Inequalities

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Solve $-4x \geq 28$

Circle your answer.

Options: $x \geq -7$, $x \leq -7$, $x \geq 7$, $x \leq 7$

Divide both sides by -4 .

Dividing by a negative number reverses the inequality sign: $\geq$ becomes $\leq$

$$28 \div (-4) = -7$$

Check with $x = -10$: $-4 \times (-10) = 40$, and $40 \geq 28$ ✓

Answer: $x \leq -7$

Tip: When you multiply or divide both sides of an inequality by a negative number, reverse the sign.

2 (1 mark)

$$-4 \leq t \leq 2$$

Circle the set of all possible values of t^2

Options: $4 \leq t^2 \leq 16$, $0 \leq t^2 \leq 16$, $-16 \leq t^2 \leq 4$, $0 \leq t^2 \leq 4$

t can be any value from -4 to 2 , including 0 .

The smallest value of t^2 is when $t = 0$: $0^2 = 0$

The largest value of t^2 is when t is furthest from 0 , at $t = -4$: $(-4)^2 = 16$

So t^2 can take any value from 0 to 16.

Answer: $0 \leq t^2 \leq 16$

Tip: Don't just square the two ends (16 and 4). $t = 0$ is allowed, so the smallest value of t^2 is 0.

3 (3 marks)

Solve $\frac{5 - 2x}{3} > \frac{x}{4} + 1$

Multiply **every** term by 12 (the lowest common multiple of 3 and 4):

$$4(5 - 2x) > 3x + 12$$

Expand: $20 - 8x > 3x + 12$

Add $8x$ to both sides: $20 > 11x + 12$

Subtract 12: $8 > 11x$

Divide by 11: $\frac{8}{11} > x$

Answer: $x < \frac{8}{11}$

Tip: The 1 must be multiplied by 12 too. Collecting the x terms on the side where they stay positive avoids dividing by a negative number.

Marks: 1 mark for multiplying every term by 12 correctly. 1 mark for collecting terms correctly, e.g. $8 > 11x$. 1 mark for $x < \frac{8}{11}$.

4 (3 marks)

Solve $(3x - 1)(x + 2) \geq 3(x^2 - 4)$

Expand the left side: $(3x - 1)(x + 2) = 3x^2 + 6x - x - 2 = 3x^2 + 5x - 2$

Expand the right side: $3(x^2 - 4) = 3x^2 - 12$

So $3x^2 + 5x - 2 \geq 3x^2 - 12$

Subtract $3x^2$ from both sides: $5x - 2 \geq -12$

Add 2: $5x \geq -10$

Divide by 5: $x \geq -2$

Answer: $x \geq -2$

Tip: The $3x^2$ terms cancel, so this is really a linear inequality. Multiply the 3 by both terms in the bracket: $3(x^2 - 4) = 3x^2 - 12$, not $3x^2 - 4$.

Marks: 1 mark for expanding both sides correctly. 1 mark for $5x \geq -10$ (or equivalent). 1 mark for $x \geq -2$.

5 (2 marks)

Work out all the integers n for which $30 - 4n^2 > 2$

Add $4n^2$ and subtract 2: $28 > 4n^2$

Divide by 4: $n^2 < 7$

$\sqrt{7} = 2.64 \dots$, so $-2.64 \dots < n < 2.64 \dots$

The integers in this range are $-2, -1, 0, 1$ and 2 .

Check $n = 3$: $30 - 36 = -6$, which is not more than 2, so 3 (and -3) do not work.

Answer: $-2, -1, 0, 1, 2$

Tip: Remember the negative integers and 0: $(-2)^2 = 4$ is less than 7 too. The question asks for the integers, so give a list, not an inequality.

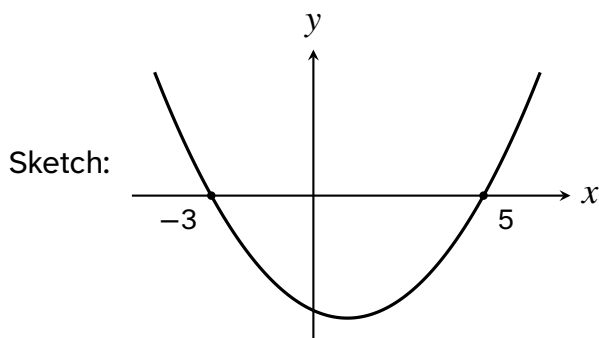
Marks: 1 mark for $n^2 < 7$ (or for $-2.6 < n < 2.6$). 1 mark for all five integers and no others.

6 (5 marks)

(a) Solve $x^2 - 2x - 15 \leq 0$

Factorise: $x^2 - 2x - 15 = (x - 5)(x + 3)$

So the curve $y = x^2 - 2x - 15$ crosses the x -axis at $x = -3$ and $x = 5$.



≤ 0 means on or below the x -axis: between the two roots, including them.

Answer: $-3 \leq x \leq 5$

Tip: A sketch tells you which part you need. Giving just the roots, $x = -3$ and $x = 5$, is not the solution of an inequality.

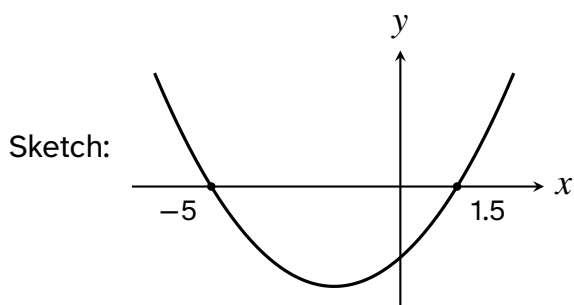
Marks: 1 mark for the critical values -3 and 5 . 1 mark for $-3 \leq x \leq 5$.

(b) Solve $2x^2 + 7x > 15$

Make one side 0: $2x^2 + 7x - 15 > 0$

Factorise: $(2x - 3)(x + 5) > 0$

Critical values: $2x - 3 = 0$ gives $x = 1.5$, and $x + 5 = 0$ gives $x = -5$



> 0 means above the x -axis: to the left of -5 and to the right of 1.5 .

Answer: $x < -5$ or $x > 1.5$

Tip: The answer has two separate parts. Don't join them into one inequality such as $-5 > x > 1.5$, which no number satisfies.

Marks: 1 mark for $2x^2 + 7x - 15$ (all terms on one side). 1 mark for factorising or for the critical values 1.5 and -5 . 1 mark for $x < -5$ or $x > 1.5$.

7 (2 marks)

A rectangle has length $(3x - 4)$ cm and width $(x + 2)$ cm.

The perimeter of the rectangle is less than 44 cm.

The length of the rectangle is greater than its width.

Work out the possible values of x .

Perimeter: $2(3x - 4) + 2(x + 2) = 6x - 8 + 2x + 4 = 8x - 4$

So $8x - 4 < 44$, which gives $8x < 48$, so $x < 6$

Length greater than width: $3x - 4 > x + 2$

Subtract x and add 4: $2x > 6$, so $x > 3$

(Both lengths are then positive, since $x > 3$ makes $3x - 4$ and $x + 2$ positive.)

Combine the two conditions: $3 < x < 6$

Answer: $3 < x < 6$

Tip: There are two conditions, so you need two inequalities. Your answer must satisfy both, so combine them into one double inequality.

Marks: 1 mark for $x < 6$ or $x > 3$ from a correct inequality. 1 mark for $3 < x < 6$.

8 (4 marks)

Given that $x > 0$, solve $2x \geq 3 + \frac{5}{x}$
You must show your working.

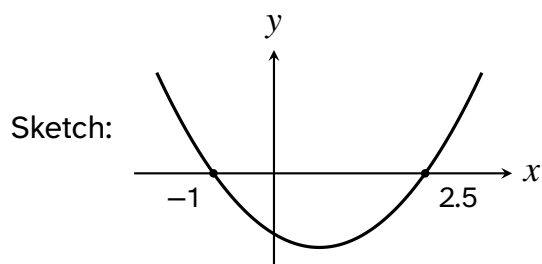
x is positive, so multiplying both sides by x does not reverse the sign.

Multiply every term by x : $2x^2 \geq 3x + 5$

Make one side 0: $2x^2 - 3x - 5 \geq 0$

Factorise: $(2x - 5)(x + 1) \geq 0$

Critical values: $x = -1$ and $x = 2.5$



≥ 0 is outside the roots: $x \leq -1$ or $x \geq 2.5$

But x is positive, so only $x \geq 2.5$ is possible.

Check: $x = 5$ gives $10 \geq 4$ ✓ $x = 1$ gives $2 \geq 8$ ✗

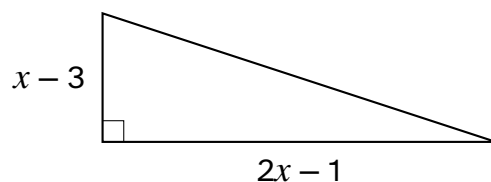
Answer: $x \geq 2.5$

Tip: Multiply **every** term by x , including the 3. At the end, use the fact that x is positive to throw away $x \leq -1$, and test a value to check your answer.

Marks: 1 mark for $2x^2 \geq 3x + 5$. 1 mark for factorising $2x^2 - 3x - 5$. 1 mark for the critical values -1 and 2.5 . 1 mark for $x \geq 2.5$ only.

9 (4 marks)

Here is a right-angled triangle with sides measured in centimetres.



Not drawn accurately

The area of the triangle is less than 9 cm^2

Use algebra to find all the possible values of x .

Do not use trial and improvement.

$$\text{Area} = \frac{1}{2} \times (2x - 1) \times (x - 3), \text{ so } \frac{1}{2}(2x - 1)(x - 3) < 9$$

$$\text{Multiply by 2: } (2x - 1)(x - 3) < 18$$

$$\text{Expand: } 2x^2 - 7x + 3 < 18$$

$$\text{Make one side 0: } 2x^2 - 7x - 15 < 0$$

$$\text{Factorise: } (2x + 3)(x - 5) < 0$$

$$\text{Critical values: } x = -1.5 \text{ and } x = 5, \text{ so } -1.5 < x < 5$$

$$\text{Lengths must be positive, so } x - 3 > 0, \text{ which gives } x > 3.$$

$$\text{(Also } 2x - 1 > 0 \text{ gives } x > 0.5, \text{ which is already true when } x > 3.)$$

Combine: $3 < x < 5$

Answer: $3 < x < 5$

Tip: Many students lose the last mark by forgetting the lower limit. Every length must be positive, so $x - 3 > 0$. The area condition only gives the upper limit.

Marks: 1 mark for $\frac{1}{2}(2x - 1)(x - 3) < 9$. 1 mark for $2x^2 - 7x - 15 < 0$. 1 mark for the upper limit $x < 5$ (from factorising). 1 mark for $3 < x < 5$.

10 (3 marks)

Sequence A has n th term $7n + 16$

Sequence B has n th term $n^2 + n$

Work out the values of n for which the n th term of sequence B is greater than the n th term of sequence A.

Do not use trial and improvement. You must show your working.

Form an inequality: $n^2 + n > 7n + 16$

Make one side 0: $n^2 - 6n - 16 > 0$

Factorise: $(n - 8)(n + 2) > 0$

Critical values: $n = 8$ and $n = -2$

Outside the roots: $n < -2$ or $n > 8$

n is a position in a sequence, so it is a positive whole number. So $n > 8$.

Check: when $n = 8$ both terms are 72 (equal, not greater). When $n = 9$, B is 90 and A is 79 ✓

Answer: $n \geq 9$ (every whole number from 9 upwards)

Tip: Solve it as an inequality, not just an equation. If you solve $n^2 - 6n - 16 = 0$, remember that at $n = 8$ the terms are equal, so the answer starts at $n = 9$.

Marks: 1 mark for $n^2 + n > 7n + 16$ (or $=$). 1 mark for $(n - 8)(n + 2)$ or the critical value 8. 1 mark for $n \geq 9$ or $n > 8$.
