

Identities and Comparing Coefficients

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Tick **one** box for each row.

	Identity	Not an identity
$(x + 5)^2 - 25 \equiv x(x + 10)$	✓	
$(2x - 1)^2 \equiv 4x^2 - 1$		✓
$x(x - 4) - 2(x - 4) \equiv (x - 2)(x - 4)$	✓	
$(x + 1)^3 \equiv x^3 + 1$		✓

An identity is true for **every** value of x : both sides expand to the same expression.

$(x + 5)^2 - 25 = x^2 + 10x + 25 - 25 = x^2 + 10x = x(x + 10)$. Identity

$(2x - 1)^2 = 4x^2 - 4x + 1$, which is not $4x^2 - 1$. Not an identity

$x(x - 4) - 2(x - 4)$ has a common factor $(x - 4)$, so it equals $(x - 2)(x - 4)$. Identity

$(x + 1)^3 = x^3 + 3x^2 + 3x + 1$, which is not $x^3 + 1$. Not an identity

Answer: Identity, Not an identity, Identity, Not an identity

Tip: To test a statement, expand or factorise one side until it matches the other. Trying one value of x can show a statement is **not** an identity, but it can never prove that it is one.

Marks: 2 marks for all four correct. 1 mark for two or three correct.

2 (1 mark)

$$2(x + a) - 3(x - 1) \equiv 9 - x$$

What is the value of a ?

Circle your answer.

Options: 3, 4.5, 6, 12

Expand the left side: $2x + 2a - 3x + 3 = -x + 2a + 3$

Compare the number terms: $2a + 3 = 9$

$$2a = 6, \text{ so } a = 3$$

Answer: 3

Tip: $-3 \times (-1) = +3$. If you write -3 here you get the wrong answer 6.

3 (3 marks)

$$a(2x + 3) - 2(x - b) \equiv 8x + 1$$

Work out the values of a and b .

Expand the left side: $2ax + 3a - 2x + 2b$

Group the terms: $(2a - 2)x + (3a + 2b)$

Compare the x terms: $2a - 2 = 8$, so $2a = 10$ and $a = 5$

Compare the number terms: $3a + 2b = 1$

$$15 + 2b = 1$$

$$2b = -14, \text{ so } b = -7$$

Answer: $a = 5$, $b = -7$

Tip: The coefficient of x on the left is $2a - 2$, not $2a$: both brackets have an x term. Write the left side as $(\)x + (\)$ before you compare.

Marks: 1 mark for expanding correctly. 1 mark for $a = 5$. 1 mark for $b = -7$.

4 (3 marks)

$$(2x - a)(bx + 3) \equiv 8x^2 + cx - 15$$

Work out the values of a , b and c .

Expand the left side: $2bx^2 + 6x - abx - 3a$

Compare the x^2 terms: $2b = 8$, so $b = 4$

Compare the number terms: $-3a = -15$, so $a = 5$

Compare the x terms: $c = 6 - ab = 6 - 5 \times 4 = -14$

Check: $(2x - 5)(4x + 3) = 8x^2 + 6x - 20x - 15 = 8x^2 - 14x - 15$ ✓

Answer: $a = 5$, $b = 4$, $c = -14$

Tip: Find a and b from the x^2 and number terms first; the x term needs both of them. Watch the signs: $-a \times 3 = -3a$.

Marks: 1 mark for $b = 4$. 1 mark for $a = 5$. 1 mark for $c = -14$.

5 (3 marks)

The coefficient of x in the expansion of $(x + k)(3x - 2)(x + 5)$ is 16

Work out the value of k .

Expand two of the brackets: $(3x - 2)(x + 5) = 3x^2 + 15x - 2x - 10 = 3x^2 + 13x - 10$

Now $(x + k)(3x^2 + 13x - 10)$

You only need the x terms:

$$x \times (-10) = -10x \quad \text{and} \quad k \times 13x = 13kx$$

So the coefficient of x is $13k - 10$

$$13k - 10 = 16$$

$$13k = 26, \text{ so } k = 2$$

Answer: $k = 2$

Tip: You do not need the whole expansion. Pick out only the products that give an x term, and remember there are two of them.

Marks: 1 mark for $3x^2 + 13x - 10$ (or another correct pair expanded). 1 mark for $13k - 10$. 1 mark for $k = 2$.

6 (3 marks)

$$(2x - 1)(x + a)^2 \equiv 2x^3 + bx^2 + cx - 9 \quad \text{where } a \text{ is a positive constant.}$$

Work out the values of a , b and c .

Compare the number terms first: the left side's number term is $-1 \times a^2 = -a^2$
 $-a^2 = -9$, so $a^2 = 9$

a is positive, so $a = 3$

$$(x + 3)^2 = x^2 + 6x + 9$$

$$\begin{aligned}(2x - 1)(x^2 + 6x + 9) &= 2x^3 + 12x^2 + 18x - x^2 - 6x - 9 \\ &= 2x^3 + 11x^2 + 12x - 9\end{aligned}$$

Compare: $b = 11$, $c = 12$

Answer: $a = 3$, $b = 11$, $c = 12$

Tip: Start with the number term: it gives a without any expanding. $a^2 = 9$ also has the solution $a = -3$, which is why the question tells you a is positive.

Marks: 1 mark for $a = 3$. 1 mark for expanding with your a . 1 mark for $b = 11$ and $c = 12$.

7 (4 marks)

$$p(x + 2)(x - 3) + q(x - 1)(x + 4) \equiv 7x^2 + 9x + r$$

Work out the values of p , q and r .

$$(x + 2)(x - 3) = x^2 - x - 6 \qquad (x - 1)(x + 4) = x^2 + 3x - 4$$

$$\text{Left side} = p(x^2 - x - 6) + q(x^2 + 3x - 4)$$

$$= (p + q)x^2 + (-p + 3q)x + (-6p - 4q)$$

$$\text{Compare the } x^2 \text{ terms: } p + q = 7$$

$$\text{Compare the } x \text{ terms: } -p + 3q = 9$$

$$\text{Add the two equations: } 4q = 16, \text{ so } q = 4$$

$$\text{Then } p = 7 - 4 = 3$$

$$\text{Compare the number terms: } r = -6p - 4q = -18 - 16 = -34$$

Answer: $p = 3, q = 4, r = -34$

Tip: Comparing coefficients often gives simultaneous equations. Label them and solve them properly. Then put your values back in to find the last unknown.

Marks: 1 mark for expanding both products. 1 mark for two correct equations in p and q . 1 mark for $p = 3$ and $q = 4$. 1 mark for $r = -34$.

8 (5 marks)

$$(2x + a)(x + b) \equiv 2x^2 + 9x + 3a - 2$$

There are two possible values of a , each with its own value of b .
Work out these values.

Do not use trial and improvement. You must show your working.

$$\text{Expand the left side: } 2x^2 + 2bx + ax + ab = 2x^2 + (a + 2b)x + ab$$

$$\text{Compare the } x \text{ terms: } a + 2b = 9, \text{ so } a = 9 - 2b$$

$$\text{Compare the number terms: } ab = 3a - 2$$

$$\text{Substitute } a = 9 - 2b: (9 - 2b)b = 3(9 - 2b) - 2$$

$$9b - 2b^2 = 27 - 6b - 2$$

$$9b - 2b^2 = 25 - 6b$$

$$\text{Rearrange: } 2b^2 - 15b + 25 = 0$$

$$\text{Factorise: } (2b - 5)(b - 5) = 0$$

$$b = 5 \text{ or } b = 2.5$$

$$\text{When } b = 5: a = 9 - 10 = -1$$

When $b = 2.5$: $a = 9 - 5 = 4$

Check: $(2x - 1)(x + 5) = 2x^2 + 9x - 5$ and $3(-1) - 2 = -5$ ✓

Answer: $a = -1, b = 5$ and $a = 4, b = 2.5$

Tip: The two equations are not both linear, so substitute one into the other to get a quadratic. Each solution of the quadratic gives a pair: match each b with its own a .

Marks: 1 mark for $a + 2b = 9$. 1 mark for $ab = 3a - 2$. 1 mark for substituting to get a quadratic in one letter. 1 mark for solving it. 1 mark for both correct pairs.
