

Functions - Domain and Range

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

$$f(x) = 4x - 1 \quad \text{for} \quad 2 \leq x \leq 5$$

Circle the range of f .

Options: $2 \leq f(x) \leq 5$, $7 \leq x \leq 19$, $7 \leq f(x) \leq 19$, $7 < f(x) < 19$

The range is the set of output values, $f(x)$.

$$f(2) = 4 \times 2 - 1 = 7$$

$$f(5) = 4 \times 5 - 1 = 19$$

The domain includes both ends ($\leq$), so the range does too.

Answer: $7 \leq f(x) \leq 19$

Tip: A range is written in terms of $f(x)$, not x . Keep $\leq$ or $<$ the same as at the matching end of the domain.

2 (4 marks)

(a) The function f is defined by $f(x) = 9 - 2x$ for $-1 < x \leq 4$

Work out the range of f .

$f(x) = 9 - 2x$ is a straight line with a negative gradient, so it goes **down** as x goes up.

$f(-1) = 9 - 2 \times (-1) = 11$ but $x = -1$ is **not** in the domain ($<$), so 11 is not in the range.

$f(4) = 9 - 2 \times 4 = 1$ and $x = 4$ **is** in the domain ($\leq$), so 1 is in the range.

Smallest output first: $1 \leq f(x) < 11$

Answer: $1 \leq f(x) < 11$

Tip: When the function is decreasing, the larger x gives the smaller output. Each inequality sign stays with its own end: $x \leq 4$ gives $1 \leq$, and $-1 < x$ gives < 11 .

Marks: 1 mark for working out 11 and 1. 1 mark for the range with the correct signs.

(b) $h(x) = 10 - 4x$

The range of h is $-6 \leq h(x) < 2$

Work out the domain of h .

Solve each end. $10 - 4x = -6$ gives $-4x = -16$, so $x = 4$

$10 - 4x = 2$ gives $-4x = -8$, so $x = 2$

$h(x) = -6$ is included, so $x = 4$ is included.

$h(x) = 2$ is not included, so $x = 2$ is not included.

Check with $x = 3$: $h(3) = -2$, which is inside the range ✓

So the domain is $2 < x \leq 4$

Answer: $2 < x \leq 4$

Tip: The domain is written in terms of x . If you solve as inequalities, dividing by -4 reverses the sign, so take care at that step.

Marks: 1 mark for $x = 4$ and $x = 2$. 1 mark for $2 < x \leq 4$.

3 (1 mark)

$$f(x) = \frac{5}{2x+3}$$

f is defined for all values of x except $x = k$

Write down the value of k .

You cannot divide by 0, so the denominator cannot be 0.

$$2x + 3 = 0$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$\text{Answer: } k = -\frac{3}{2}$$

4 (2 marks)

$$f(x) = (x + 1)^2 - 4$$

The domain of f is $-3 < x \leq 2$

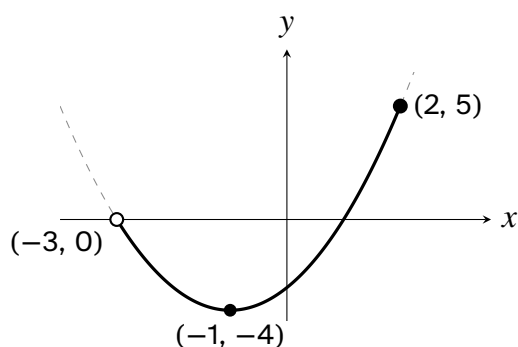
Work out the range of f .

The graph is a U-shaped curve with its minimum point at $(-1, -4)$.

$x = -1$ is inside the domain, so the smallest output is -4 .

Now check both ends of the domain:

$$f(-3) = (-2)^2 - 4 = 0 \quad f(2) = 3^2 - 4 = 5$$



The largest output is 5 (at $x = 2$, which is included).

The smallest output is -4 (at $x = -1$, which is included).

$x = -3$ is not included, but the output 0 still happens elsewhere in the domain, so it makes no difference.

$$\text{Answer: } -4 \leq f(x) \leq 5$$

Tip: Don't just put the two ends of the domain into the function. If the turning point is inside the domain, the minimum output is there. A quick sketch shows this.

Marks: 1 mark for -4 or 5 as one end. 1 mark for the correct range.

5 (2 marks)

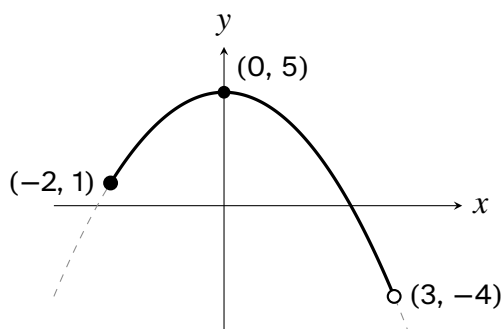
$$g(x) = 5 - x^2 \quad \text{for} \quad -2 \leq x < 3$$

Work out the range of g .

The graph is an upside-down U with its maximum at $(0, 5)$.

$x = 0$ is inside the domain, so the largest output is 5.

$$g(-2) = 5 - 4 = 1 \quad g(3) = 5 - 9 = -4$$



The smallest output is -4 , at $x = 3$, but $x = 3$ is **not** in the domain.

So -4 is not reached: $-4 < g(x)$

Range: $-4 < g(x) \leq 5$

Answer: $-4 < g(x) \leq 5$

Tip: The maximum of 5 comes from the turning point, not from either end of the domain.

Answers of $-4 < g(x) \leq 1$ miss this.

Marks: 1 mark for 5 and -4 as the ends. 1 mark for the correct signs.

6 (2 marks)

A function h has domain $-2 \leq x < 3$, where $h(x) = 1 - x^3$

Work out the range of h .

x^3 always increases as x increases, so $1 - x^3$ always **decreases**.

So the ends of the domain give the ends of the range.

$$h(-2) = 1 - (-2)^3 = 1 - (-8) = 9 \quad (\text{included})$$

$$h(3) = 1 - 3^3 = 1 - 27 = -26 \quad (\text{not included})$$

Smallest first: $-26 < h(x) \leq 9$

Answer: $-26 < h(x) \leq 9$

Tip: Bracket negative numbers when you substitute: $(-2)^3 = -8$, and $1 - (-8) = 9$. Write the answer with $h(x)$ in the middle, not x .

Marks: 1 mark for 9 and -26 . 1 mark for the range with the correct signs.

7 (2 marks)

$f(x) = 2x^2 - 1$, where $x > 0$

For a smaller domain, the range of f is $7 < f(x) \leq 49$

Work out this domain.

For $x > 0$, $2x^2 - 1$ increases as x increases.

Lower end: $2x^2 - 1 = 7$, so $2x^2 = 8$, $x^2 = 4$, $x = 2$ (positive)

Upper end: $2x^2 - 1 = 49$, so $2x^2 = 50$, $x^2 = 25$, $x = 5$ (positive)

7 is not included, so $x = 2$ is not included. 49 is included, so $x = 5$ is included.

Domain: $2 < x \leq 5$

Answer: $2 < x \leq 5$

Tip: $x > 0$, so ignore the negative square roots (-2 and -5).

Marks: 1 mark for $x = 2$ and $x = 5$. 1 mark for $2 < x \leq 5$.

8 (4 marks)

$h(x) = a \times b^{-x}$ for $-2 \leq x \leq 3$

a and b are positive constants.

The graph of $y = h(x)$ passes through $(-1, 24)$ and $(2, 3)$

Work out the range of h .

Substitute each point into $y = a \times b^{-x}$:

$(-1, 24)$: $24 = a \times b^1 = ab$

$$(2, 3): \quad 3 = a \times b^{-2} = \frac{a}{b^2}$$

Divide the first equation by the second: $\frac{ab}{a/b^2} = \frac{24}{3}$, so $b^3 = 8$

$$b = 2, \text{ and then } a = 24 \div 2 = 12$$

So $h(x) = 12 \times 2^{-x}$, which decreases as x increases.

$$h(-2) = 12 \times 2^2 = 48$$

$$h(3) = 12 \times 2^{-3} = \frac{12}{8} = 1.5$$

Both ends are included: $1.5 \leq h(x) \leq 48$

Answer: $1.5 \leq h(x) \leq 48$

Tip: $a \times b^{-x}$ means a times b^{-x} , not $(ab)^{-x}$. The range is the set of outputs, so don't subtract the ends (that is a statistical range).

Marks: 1 mark for one correct equation in a and b . 1 mark for $b = 2$. 1 mark for $a = 12$. 1 mark for the range.

9 (3 marks)

$f(x) = x^2 - 8x + k$ for $1 \leq x \leq 10$, where k is a constant.

The range of f is $3 \leq f(x) \leq q$

Work out the values of k and q .

Complete the square: $x^2 - 8x + k = (x - 4)^2 - 16 + k$

The minimum point is at $x = 4$, which is inside the domain.

So the smallest output is $k - 16$, and this must be 3.

$$k - 16 = 3, \text{ so } k = 19$$

Now $f(x) = (x - 4)^2 + 3$. Check both ends of the domain:

$$f(1) = (-3)^2 + 3 = 12 \quad f(10) = 6^2 + 3 = 39$$

The largest output is 39, so $q = 39$

Answer: $k = 19$, $q = 39$

Tip: The lowest output is at the turning point, not at $x = 1$. For the highest output, try both ends of the domain: $x = 10$ is further from $x = 4$, so it gives the larger value.

Marks: 1 mark for the minimum at $x = 4$ (or $k - 16$). 1 mark for $k = 19$. 1 mark for $q = 39$.
