

Factorising

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Factorise fully

(a) $20p^2 - 15p^3$

The highest common factor of 20 and 15 is 5.

The highest power of p in both terms is p^2 .

So take out $5p^2$:

$$20p^2 \div 5p^2 = 4 \quad \text{and} \quad 15p^3 \div 5p^2 = 3p$$

Answer: $5p^2(4 - 3p)$

Tip: “Fully” means the highest common factor: both the number **and** the letter.

$5p(4p - 3p^2)$ still has a common factor of p inside the bracket.

Marks: 1 mark for a correct partial factorisation, e.g. $5p(4p - 3p^2)$ or $p^2(20 - 15p)$. 1 mark for the correct answer.

(b) $8m^2n^3 + 12mn^2$

The highest common factor of 8 and 12 is 4.

Both terms contain m (lowest power m^1) and n (lowest power n^2).

So take out $4mn^2$:

$$8m^2n^3 \div 4mn^2 = 2mn \quad \text{and} \quad 12mn^2 \div 4mn^2 = 3$$

Check: $4mn^2(2mn + 3) = 8m^2n^3 + 12mn^2$ ✓

Answer: $4mn^2(2mn + 3)$

Tip: Check each letter separately. Take out the **lowest** power of each letter that appears in every term.

Marks: 1 mark for taking out any two of 4, m and n^2 correctly. 1 mark for the correct answer.

2 (1 mark)

Which of these is a factorisation of $16x^2 - 81y^2$?

Circle your answer.

Options: $(4x - 9y)^2$, $(4x + 9y)(4x - 9y)$, $(8x + 9y)(2x - 9y)$, $(16x - 81y)(x + y)$

$$16x^2 = (4x)^2 \text{ and } 81y^2 = (9y)^2$$

So this is a difference of two squares: $(4x)^2 - (9y)^2$

Use $a^2 - b^2 = (a + b)(a - b)$.

Answer: $(4x + 9y)(4x - 9y)$

Tip: Look out for a difference of two squares: two square terms with a minus sign between them. $(4x - 9y)^2$ would give a middle term of $-72xy$.

3 (4 marks)

Factorise

(a) $5x^2 + 17x + 6$

Multiply the first and last numbers: $5 \times 6 = 30$

Find two numbers that multiply to 30 and add to 17: 15 and 2

Split the middle term: $5x^2 + 15x + 2x + 6$

Factorise in pairs: $5x(x + 3) + 2(x + 3)$

Take out $(x + 3)$.

Answer: $(5x + 2)(x + 3)$

Marks: 1 mark for $(5x \pm 2)(x \pm 3)$ or a correct split of the middle term. 1 mark for the correct answer.

(b) $10x^2 - 11xy - 6y^2$

This works just like a quadratic in x , with y in each bracket.

Multiply the first and last coefficients: $10 \times (-6) = -60$

Find two numbers that multiply to -60 and add to -11 : -15 and 4

Split the middle term: $10x^2 - 15xy + 4xy - 6y^2$

Factorise in pairs: $5x(2x - 3y) + 2y(2x - 3y)$

Take out $(2x - 3y)$.

Check: $(5x + 2y)(2x - 3y) = 10x^2 - 15xy + 4xy - 6y^2 = 10x^2 - 11xy - 6y^2$ ✓

Answer: $(5x + 2y)(2x - 3y)$

Tip: Every term in the brackets needs its letter: $(5x + 2y)(2x - 3y)$, not $(5x + 2)(2x - 3)$. Expand your answer to check the signs.

Marks: 1 mark for brackets that give two of the three terms correctly. 1 mark for the correct answer.

4 (5 marks)

Factorise fully

(a) $18y^2 - 32$

Both numbers are even, so take out the common factor 2:

$$18y^2 - 32 = 2(9y^2 - 16)$$

$9y^2 - 16 = (3y)^2 - 4^2$ is a difference of two squares.

$$\text{So } 9y^2 - 16 = (3y + 4)(3y - 4)$$

Keep the 2 at the front.

Answer: $2(3y + 4)(3y - 4)$

Tip: This is an expression, not an equation, so you cannot divide the 2 away. It must stay in your answer. Don't try to solve it either.

Marks: 1 mark for $2(9y^2 - 16)$ or $(6y + 8)(3y - 4)$. 1 mark for the correct answer.

(b) $2x^5 - 32x$

Take out the common factor $2x$:

$$2x^5 - 32x = 2x(x^4 - 16)$$

$x^4 - 16 = (x^2)^2 - 4^2$ is a difference of two squares:

$$x^4 - 16 = (x^2 + 4)(x^2 - 4)$$

$x^2 - 4 = x^2 - 2^2$ is another difference of two squares:

$$x^2 - 4 = (x + 2)(x - 2)$$

$x^2 + 4$ is a sum of two squares, so it does not factorise.

Answer: $2x(x^2 + 4)(x + 2)(x - 2)$

Tip: Keep going until no bracket will factorise any further. $x^2 - 4$ still factorises; $x^2 + 4$ does not.

Marks: 1 mark for $2x(x^4 - 16)$. 1 mark for $(x^2 + 4)(x^2 - 4)$. 1 mark for the fully factorised answer.

5 (3 marks)

Factorise fully $(5x - 1)^2 - (x + 3)^2$

This is a difference of two squares, $a^2 - b^2$, with $a = 5x - 1$ and $b = x + 3$.

$$a^2 - b^2 = (a - b)(a + b)$$

$$a - b = (5x - 1) - (x + 3) = 5x - 1 - x - 3 = 4x - 4$$

$$a + b = (5x - 1) + (x + 3) = 6x + 2$$

So the expression is $(4x - 4)(6x + 2)$.

Each bracket has a common factor: $4x - 4 = 4(x - 1)$ and $6x + 2 = 2(3x + 1)$

$$4 \times 2 = 8$$

Answer: $8(x - 1)(3x + 1)$

Tip: Put brackets round $(x + 3)$ when you subtract it: $-(x + 3) = -x - 3$. Then check each bracket for a common factor, or you have not factorised fully.

Marks: 1 mark for $(5x - 1 - x - 3)$ or $(5x - 1 + x + 3)$, or for expanding to $24x^2 - 16x - 8$. 1 mark for $(4x - 4)(6x + 2)$. 1 mark for the correct answer.

6 (3 marks)

Factorise fully $x(x + 5)^4 - 6(x + 5)^3$

Both terms contain $(x + 5)^3$, so take it out as a common factor:

$$x(x + 5)^4 \div (x + 5)^3 = x(x + 5)$$

$$6(x + 5)^3 \div (x + 5)^3 = 6$$

So the expression is $(x + 5)^3[x(x + 5) - 6]$

Simplify the square bracket: $x^2 + 5x - 6$

Factorise it: two numbers that multiply to -6 and add to 5 are 6 and -1 .

$$x^2 + 5x - 6 = (x + 6)(x - 1)$$

Answer: $(x + 5)^3(x + 6)(x - 1)$

Tip: Don't expand the powers: take out the highest power of $(x + 5)$ that is in both terms. Then simplify and check whether what is left factorises again.

Marks: 1 mark for taking out $(x + 5)^3$. 1 mark for $(x + 5)^3(x^2 + 5x - 6)$. 1 mark for the fully factorised answer.

7 (3 marks)

Solve $6x^3 + 7x^2 - 20x = 0$

You must show your working.

Every term contains x , so take out the common factor x :

$$x(6x^2 + 7x - 20) = 0$$

Factorise the quadratic: $6 \times (-20) = -120$, and 15 and -8 multiply to -120 and add to 7.

$$6x^2 + 15x - 8x - 20 = 3x(2x + 5) - 4(2x + 5) = (3x - 4)(2x + 5)$$

$$\text{So } x(3x - 4)(2x + 5) = 0$$

One of the factors must be 0:

$$x = 0, \quad 3x - 4 = 0 \text{ so } x = \frac{4}{3}, \quad 2x + 5 = 0 \text{ so } x = -\frac{5}{2}$$

Answer: $x = 0, x = \frac{4}{3}, x = -\frac{5}{2}$

Tip: Don't divide both sides by x : that loses the solution $x = 0$. And don't stop at the brackets; "solve" means write down the values of x .

Marks: 1 mark for $x(6x^2 + 7x - 20)$. 1 mark for $x(3x - 4)(2x + 5)$. 1 mark for all three solutions.

8 (3 marks)

Solve $2\cos^2 x + \cos x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$

You must show your working.

This is a quadratic in $\cos x$. It factorises like $2y^2 + y - 1 = (2y - 1)(y + 1)$.

$$\text{So } (2\cos x - 1)(\cos x + 1) = 0$$

$$\text{So } \cos x = \frac{1}{2} \text{ or } \cos x = -1$$

$$\cos x = \frac{1}{2} \text{ gives } x = 60^\circ, \text{ and from the symmetry of the cosine graph} \\ x = 360^\circ - 60^\circ = 300^\circ$$

$\cos x = -1$ gives $x = 180^\circ$ (the minimum of the cosine graph)

Answer: $x = 60^\circ, 180^\circ, 300^\circ$

Tip: Treat $\cos x$ like a single letter and factorise as normal. Then solve each bracket: $\cos x = \frac{1}{2}$ has two answers between 0° and 360° , but $\cos x = -1$ has only one.

Marks: 1 mark for $(2 \cos x - 1)(\cos x + 1) = 0$. 1 mark for $\cos x = \frac{1}{2}$ and $\cos x = -1$. 1 mark for all three angles and no extras.
