

Expanding Brackets

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Expand and simplify

(a) $(4x - 3y)(2x + 5y)$

Multiply each term in the first bracket by each term in the second:

$$4x \times 2x = 8x^2 \quad 4x \times 5y = 20xy$$

$$-3y \times 2x = -6xy \quad -3y \times 5y = -15y^2$$

Collect the xy terms: $20xy - 6xy = 14xy$

Answer: $8x^2 + 14xy - 15y^2$

Tip: $-3y \times 2x$ can be written $-6xy$ or $-6yx$: they are like terms with $20xy$, so collect them to get $14xy$.

Marks: 1 mark for four terms with at least three correct. 1 mark for the fully simplified answer.

(b) $(x^2 - 4y)(3x + 2y)$

$$x^2 \times 3x = 3x^3 \quad x^2 \times 2y = 2x^2y$$

$$-4y \times 3x = -12xy \quad -4y \times 2y = -8y^2$$

None of these four terms are like terms, so nothing collects.

Answer: $3x^3 + 2x^2y - 12xy - 8y^2$

Tip: When the four terms are all different, the answer has four terms. Do not force terms such as $2x^2y$ and $-12xy$ together.

Marks: 1 mark for three correct terms. 1 mark for all four correct.

2 (1 mark)

Circle the expansion of $(3 - 2x)^2$

Options: $9 - 4x^2$, $9 + 4x^2$, $9 - 6x + 4x^2$, $9 - 12x + 4x^2$

$$(3 - 2x)^2 = (3 - 2x)(3 - 2x)$$

$$= 9 - 6x - 6x + 4x^2$$

$$= 9 - 12x + 4x^2$$

Answer: $9 - 12x + 4x^2$

Tip: Squaring a bracket gives a middle term: here $2 \times 3 \times (-2x) = -12x$. Also $(-2x)^2 = +4x^2$, not $-4x^2$.

3 (3 marks)

Expand and simplify $(2x^2 + x - 4)(3x - 2)$

Multiply each of the three terms by $3x$ and then by -2 :

$$2x^2 \times 3x = 6x^3 \quad 2x^2 \times (-2) = -4x^2$$

$$x \times 3x = 3x^2 \quad x \times (-2) = -2x$$

$$-4 \times 3x = -12x \quad -4 \times (-2) = +8$$

$$\text{So we get } 6x^3 - 4x^2 + 3x^2 - 2x - 12x + 8$$

Collect like terms: $-4x^2 + 3x^2 = -x^2$ and $-2x - 12x = -14x$

Answer: $6x^3 - x^2 - 14x + 8$

Tip: A three-term bracket times a two-term bracket gives $3 \times 2 = 6$ terms before you simplify. Count them to check none are missing.

Marks: 1 mark for six terms with at least four correct. 1 mark for all six correct. 1 mark for the simplified answer.

4 (6 marks)

Expand and simplify fully

(a) $(5 - 2x)(3x + 1)(4x - 3)$

Expand the first two brackets:

$$(5 - 2x)(3x + 1) = 15x + 5 - 6x^2 - 2x = -6x^2 + 13x + 5$$

Now multiply by $(4x - 3)$:

$$-6x^2 \times 4x = -24x^3 \quad -6x^2 \times (-3) = 18x^2$$

$$13x \times 4x = 52x^2 \quad 13x \times (-3) = -39x$$

$$5 \times 4x = 20x \quad 5 \times (-3) = -15$$

Collect like terms: $18x^2 + 52x^2 = 70x^2$ and $-39x + 20x = -19x$

Answer: $-24x^3 + 70x^2 - 19x - 15$

Tip: The x^3 term is negative here, and that is fine. Do not change the signs of a correct answer to make the first term positive.

Marks: 1 mark for expanding two brackets correctly. 1 mark for six terms with at least four correct. 1 mark for the correct answer.

(b) $(2x - 5)^2(x + 4)$

Expand the square first:

$$(2x - 5)^2 = (2x - 5)(2x - 5) = 4x^2 - 10x - 10x + 25 = 4x^2 - 20x + 25$$

Now multiply by $(x + 4)$:

$$4x^2 \times x = 4x^3 \quad 4x^2 \times 4 = 16x^2$$

$$-20x \times x = -20x^2 \quad -20x \times 4 = -80x$$

$$25 \times x = 25x \quad 25 \times 4 = 100$$

Collect like terms: $16x^2 - 20x^2 = -4x^2$ and $-80x + 25x = -55x$

Answer: $4x^3 - 4x^2 - 55x + 100$

Tip: $(2x - 5)^2$ is not $4x^2 - 25$ or $4x^2 + 25$. Write it as $(2x - 5)(2x - 5)$ so you get the middle term $-20x$.

Marks: 1 mark for $4x^2 - 20x + 25$. 1 mark for six terms with at least four correct. 1 mark for the correct answer.

5 (3 marks)

Expand and simplify $\left(x + \frac{3}{x}\right)\left(2x - \frac{1}{x}\right)$

Multiply each term in the first bracket by each term in the second:

$$x \times 2x = 2x^2$$

$$x \times \left(-\frac{1}{x}\right) = -1$$

$$\frac{3}{x} \times 2x = 6$$

$$\frac{3}{x} \times \left(-\frac{1}{x}\right) = -\frac{3}{x^2}$$

Collect the number terms: $-1 + 6 = 5$

Answer: $2x^2 + 5 - \frac{3}{x^2}$

Tip: $x \times \frac{1}{x} = 1$, so two of the four products are just numbers. Collect them. The last term can also be written as $-3x^{-2}$.

Marks: 1 mark for three of the four products correct. 1 mark for all four correct. 1 mark for the simplified answer.

6 (4 marks)

Expand and simplify fully $(4x^2 - 1)(x + 2) - (2x + 1)(2x^2 - 3x + 5)$

Expand the first product:

$$(4x^2 - 1)(x + 2) = 4x^3 + 8x^2 - x - 2$$

Expand the second product:

$$\begin{aligned}(2x + 1)(2x^2 - 3x + 5) &= 4x^3 - 6x^2 + 10x + 2x^2 - 3x + 5 \\ &= 4x^3 - 4x^2 + 7x + 5\end{aligned}$$

Subtract the **whole** of the second product (keep it in a bracket):

$$\begin{aligned}4x^3 + 8x^2 - x - 2 - (4x^3 - 4x^2 + 7x + 5) \\ = 4x^3 + 8x^2 - x - 2 - 4x^3 + 4x^2 - 7x - 5\end{aligned}$$

The x^3 terms cancel: $4x^3 - 4x^3 = 0$

$$8x^2 + 4x^2 = 12x^2, \quad -x - 7x = -8x, \quad -2 - 5 = -7$$

Answer: $12x^2 - 8x - 7$

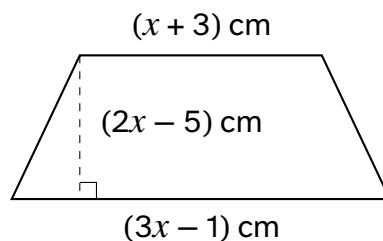
Tip: The minus sign applies to every term of the second product. Put it in a bracket first, then change all the signs: $-(4x^3 - 4x^2 + 7x + 5) = -4x^3 + 4x^2 - 7x - 5$.

Marks: 1 mark for $4x^3 + 8x^2 - x - 2$. 1 mark for $4x^3 - 4x^2 + 7x + 5$. 1 mark for subtracting all four terms of the second product. 1 mark for the correct answer.

7 (3 marks)

The cross-section of a prism is a trapezium.

The prism has length $(x + 2)$ cm.



Not drawn accurately

The volume of the prism is $V \text{ cm}^3$

Show that $V = 4x^3 - 21x - 10$

$$\text{Area of a trapezium} = \frac{1}{2}(a + b)h$$

$$a + b = (x + 3) + (3x - 1) = 4x + 2$$

$$\text{Area} = \frac{1}{2}(4x + 2)(2x - 5) = (2x + 1)(2x - 5)$$

$$= 4x^2 - 10x + 2x - 5 = 4x^2 - 8x - 5$$

Volume = area of cross-section $\times$ length

$$V = (4x^2 - 8x - 5)(x + 2)$$

$$= 4x^3 + 8x^2 - 8x^2 - 16x - 5x - 10$$

$$= 4x^3 - 21x - 10, \text{ as required.}$$

Answer: Shown: $V = (4x^2 - 8x - 5)(x + 2) = 4x^3 - 21x - 10$

Tip: Halve the $(4x + 2)$ first to get $(2x + 1)$: it keeps the numbers small. In a “show that” question, write every line of the expansion so the examiner can see how you reach the given answer.

Marks: 1 mark for the area of the cross-section, $4x^2 - 8x - 5$ (or unsimplified). 1 mark for multiplying by $(x + 2)$ with at least four correct terms. 1 mark for a complete, correct expansion.

8 (3 marks)

Expand and simplify $(2x - y)(2x + y)(4x^2 + y^2)$

Multiply the first two brackets:

$$(2x - y)(2x + y) = 4x^2 + 2xy - 2xy - y^2 = 4x^2 - y^2$$

Now multiply by $(4x^2 + y^2)$:

$$(4x^2 - y^2)(4x^2 + y^2) = 16x^4 + 4x^2y^2 - 4x^2y^2 - y^4$$

The x^2y^2 terms cancel.

Answer: $16x^4 - y^4$

Tip: Choose the order: $(2x - y)(2x + y)$ is a difference of two squares, so the middle terms cancel straight away. Starting with any other pair works but gives many more terms.

Marks: 1 mark for $4x^2 - y^2$. 1 mark for four terms from the next product. 1 mark for the correct answer.

9 (3 marks)

Prove that $(3n + 4)^2 - (3n - 2)(3n + 2)$ is a multiple of 4 for all integer values of n .

$$(3n + 4)^2 = (3n + 4)(3n + 4) = 9n^2 + 12n + 12n + 16 = 9n^2 + 24n + 16$$

$$(3n - 2)(3n + 2) = 9n^2 + 6n - 6n - 4 = 9n^2 - 4$$

$$\text{Subtract: } 9n^2 + 24n + 16 - (9n^2 - 4)$$

$$= 9n^2 + 24n + 16 - 9n^2 + 4$$

$$= 24n + 20$$

$$= 4(6n + 5)$$

$6n + 5$ is an integer, so $4(6n + 5)$ is a multiple of 4.

Answer: $24n + 20 = 4(6n + 5)$, which is a multiple of 4

Tip: Finish with a statement. Writing $24n + 20$ is not enough: factorise to $4(6n + 5)$ and say why this is a multiple of 4. Remember the bracket when you subtract $9n^2 - 4$, so $-(-4) = +4$.

Marks: 1 mark for $9n^2 + 24n + 16$ or $9n^2 - 4$. 1 mark for $24n + 20$. 1 mark for $4(6n + 5)$ with a conclusion.
