

Equations of Straight Lines

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Circle the gradient of the line $5x + 2y = 8$

Options: $-\frac{5}{2}$, $-\frac{2}{5}$, $\frac{5}{2}$, 5

Make y the subject so the equation is in the form $y = mx + c$.

Subtract $5x$: $2y = -5x + 8$

Divide every term by 2: $y = -\frac{5}{2}x + 4$

The gradient is the number in front of x .

Answer: $-\frac{5}{2}$

Tip: You can only read the gradient off when the equation starts " $y =$ ". The gradient is not 5, and it is negative because $5x$ moves to the other side.

2 (2 marks)

Work out the gradient of the straight line through the points $(-3, 8)$ and $(5, -4)$

Gradient = $\frac{\text{change in } y}{\text{change in } x}$

Change in y : $-4 - 8 = -12$

Change in x : $5 - (-3) = 8$

Gradient = $\frac{-12}{8} = -\frac{3}{2}$

The line goes down from left to right, so a negative gradient makes sense.

Answer: $-\frac{3}{2}$

Tip: Put the change in y on top. Take care with the negatives: $5 - (-3) = 8$, and subtract the coordinates in the same order on the top and the bottom.

Marks: 1 mark for $-\frac{12}{8}$ (or $\frac{12}{-8}$). 1 mark for $-\frac{3}{2}$ (or -1.5).

3 (3 marks)

Line K has equation $3y = 2x - 12$

- (a) K crosses the y -axis at the point A .
Write down the coordinates of A .

On the y -axis, $x = 0$.

$$3y = 2 \times 0 - 12 = -12$$

$$y = -4$$

Answer: $(0, -4)$

Tip: The y -intercept is not -12 . Divide every term by 3 first ($y = \frac{2}{3}x - 4$), or simply put $x = 0$.

- (b) On the grid, draw line K for values of x from -3 to 9

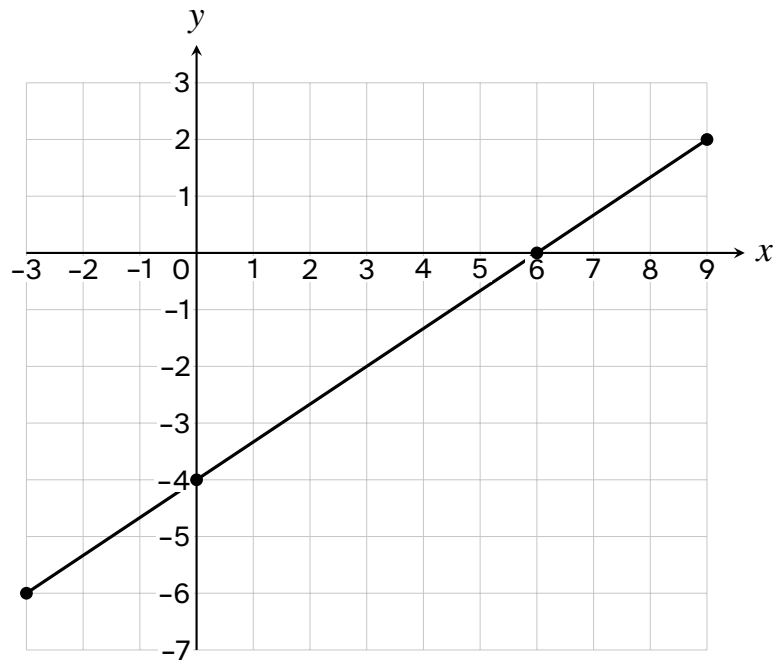
Work out some points on the line (choose values of x that are multiples of 3):

$$x = -3: 3y = -6 - 12 = -18, \text{ so } y = -6$$

$$x = 0: y = -4 \quad x = 6: 3y = 0, \text{ so } y = 0$$

$$x = 9: 3y = 18 - 12 = 6, \text{ so } y = 2$$

Plot the points and join them with a ruler from $x = -3$ to $x = 9$.



Tip: Work out three points: if they do not line up, there is a mistake. Use a ruler and draw the line across the **whole** range asked for, from $x = -3$ to $x = 9$.

Marks: 1 mark for at least two correct points plotted. 1 mark for a correct straight line from $x = -3$ to $x = 9$.

4 (6 marks)

The straight line L has equation $2x - 5y = 3$

- (a) Work out the equation of the line that is parallel to L and passes through the point $(10, -1)$

Give your answer in the form $y = mx + c$

Rearrange L into the form $y = mx + c$:

$$2x - 5y = 3 \text{ gives } 5y = 2x - 3$$

$$\text{So } y = \frac{2}{5}x - \frac{3}{5}, \text{ and the gradient of } L \text{ is } \frac{2}{5}$$

Parallel lines have the same gradient: $y = \frac{2}{5}x + c$

$$\text{Substitute } (10, -1): -1 = \frac{2}{5} \times 10 + c$$

$$-1 = 4 + c, \text{ so } c = -5$$

Answer: $y = \frac{2}{5}x - 5$

Tip: Move the y term so it is positive ($5y = 2x - 3$), then divide **every** term by 5. The gradient is $\frac{2}{5}$, not 2 or $-\frac{2}{5}$.

Marks: 1 mark for the gradient $\frac{2}{5}$. 1 mark for substituting $(10, -1)$ with this gradient. 1 mark for $y = \frac{2}{5}x - 5$.

- (b) Work out the equation of the line that is perpendicular to L and passes through the point where L crosses the x -axis.
Give your answer in the form $ax + by = c$ where a, b and c are integers.

L crosses the x -axis where $y = 0$: $2x = 3$, so $x = \frac{3}{2}$

The point is $\left(\frac{3}{2}, 0\right)$

Perpendicular gradient: flip $\frac{2}{5}$ and change the sign, so $-\frac{5}{2}$

Use $y - y_1 = m(x - x_1)$: $y - 0 = -\frac{5}{2}\left(x - \frac{3}{2}\right)$

$$y = -\frac{5}{2}x + \frac{15}{4}$$

Multiply every term by 4: $4y = -10x + 15$

Add $10x$ to both sides: $10x + 4y = 15$

Answer: $10x + 4y = 15$

Tip: On the x -axis $y = 0$, not $x = 0$. To clear the fractions $\frac{5}{2}$ and $\frac{15}{4}$, multiply **every** term by 4.

Marks: 1 mark for $\left(\frac{3}{2}, 0\right)$. 1 mark for the gradient $-\frac{5}{2}$ used with this point. 1 mark for $10x + 4y = 15$ (or any integer multiple of it).

5 (3 marks)

Here are the equations of three straight lines.

Line *A*: $4y = 3x + 5$

Line *B*: $3x + 4y = 12$

Line *C*: $8x + 6y = 1$

Two of the lines are perpendicular.

Which two?

You must show your working.

Rearrange each line into the form $y = mx + c$.

Line *A*: $y = \frac{3}{4}x + \frac{5}{4}$, so the gradient is $\frac{3}{4}$

Line *B*: $4y = -3x + 12$, $y = -\frac{3}{4}x + 3$, so the gradient is $-\frac{3}{4}$

Line *C*: $6y = -8x + 1$, $y = -\frac{4}{3}x + \frac{1}{6}$, so the gradient is $-\frac{4}{3}$

Perpendicular gradients multiply to -1 :

A and *C*: $\frac{3}{4} \times \left(-\frac{4}{3}\right) = -1$ ✓

(*A* and *B* give $-\frac{9}{16}$, and *B* and *C* give 1.)

Answer: Lines *A* and *C*

Tip: $-\frac{3}{4}$ and $-\frac{4}{3}$ are flipped but have the same sign, so *B* and *C* are not perpendicular. A perpendicular gradient is flipped **and** changes sign.

Marks: 1 mark for two correct gradients. 1 mark for all three gradients. 1 mark for *A* and *C*, with the product -1 shown.

6 (4 marks)

Line *L* passes through the points $(-2, 9)$ and $(4, -3)$

Line *M* has equation $x - 3y = 6$

Work out the coordinates of the point where *L* and *M* intersect.

You must show your working.

Gradient of *L* = $\frac{-3 - 9}{4 - (-2)} = \frac{-12}{6} = -2$

So L is $y = -2x + c$

Substitute $(-2, 9)$: $9 = -2 \times (-2) + c$, so $c = 5$

L : $y = -2x + 5$

Substitute this into M : $x - 3(-2x + 5) = 6$

$$x + 6x - 15 = 6$$

$$7x = 21, \text{ so } x = 3$$

$$y = -2 \times 3 + 5 = -1$$

Check in M : $3 - 3 \times (-1) = 6$ ✓

Answer: $(3, -1)$

Tip: Bracket the expression when you substitute: $-3(-2x + 5) = 6x - 15$. Check your point in **both** equations.

Marks: 1 mark for the gradient -2 . 1 mark for $y = -2x + 5$. 1 mark for an equation in one unknown. 1 mark for $(3, -1)$.

7 (4 marks)

The lines $3x + 2y = 18$ and $y = mx - 4$ meet at a point on the x -axis.
Show that the two lines are perpendicular.

On the x -axis, $y = 0$.

$$3x + 2 \times 0 = 18, \text{ so } x = 6. \text{ The lines meet at } (6, 0).$$

$$(6, 0) \text{ is on } y = mx - 4: 0 = 6m - 4, \text{ so } m = \frac{2}{3}$$

Gradient of $3x + 2y = 18$: $2y = -3x + 18$, $y = -\frac{3}{2}x + 9$, so the gradient is $-\frac{3}{2}$

$$\frac{2}{3} \times \left(-\frac{3}{2}\right) = -1$$

The gradients multiply to -1 , so the lines are perpendicular.

Answer: $m = \frac{2}{3}$ and the gradients $\frac{2}{3}$ and $-\frac{3}{2}$ multiply to -1 , so the lines are perpendicular

Tip: The point on the x -axis has $y = 0$. In a “show that” you must finish with a statement, such as “the gradients multiply to -1 , so the lines are perpendicular”.

Marks: 1 mark for $(6, 0)$. 1 mark for $m = \frac{2}{3}$. 1 mark for the gradient $-\frac{3}{2}$. 1 mark for showing the product is -1 with a conclusion.

8 (5 marks)

The straight line L passes through the points $A(\sqrt{2}, 5)$ and $B(3\sqrt{2}, 1)$

(a) Show that the equation of L can be written as $y = 7 - \sqrt{2}x$

$$\text{Gradient} = \frac{1 - 5}{3\sqrt{2} - \sqrt{2}} = \frac{-4}{2\sqrt{2}} = -\frac{2}{\sqrt{2}}$$

$$\text{Rationalise: } -\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

$$\text{So } y = -\sqrt{2}x + c$$

$$\text{Substitute } A(\sqrt{2}, 5): 5 = -\sqrt{2} \times \sqrt{2} + c$$

$$\sqrt{2} \times \sqrt{2} = 2, \text{ so } 5 = -2 + c \text{ and } c = 7$$

$$\text{So the equation is } y = 7 - \sqrt{2}x$$

Answer: Gradient $-\sqrt{2}$ and $c = 7$, so $y = 7 - \sqrt{2}x$

Tip: Collect like surds: $3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$. Remember $\sqrt{2} \times \sqrt{2} = 2$.

Marks: 1 mark for the gradient as $\frac{-4}{2\sqrt{2}}$ or better. 1 mark for simplifying it to $-\sqrt{2}$. 1 mark for $c = 7$ shown.

(b) L crosses the x -axis at the point C .

Work out the x -coordinate of C .

Give your answer in the form $\frac{a\sqrt{2}}{b}$ where a and b are integers.

$$\text{On the } x\text{-axis, } y = 0: 0 = 7 - \sqrt{2}x$$

$$\sqrt{2}x = 7, \text{ so } x = \frac{7}{\sqrt{2}}$$

$$\text{Rationalise: } \frac{7}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$$

Answer: $\frac{7\sqrt{2}}{2}$

Tip: Give the answer in the form asked: $\frac{7}{\sqrt{2}}$ still has a surd on the bottom, so rationalise it.

Marks: 1 mark for $\frac{7}{\sqrt{2}}$. 1 mark for $\frac{7\sqrt{2}}{2}$.

9 (4 marks)

Line P has equation $y = kx - 3$

Line Q has equation $(k + 5)x + 6y = 1$

k is a constant.

Lines P and Q are perpendicular.

Work out the two possible values of k .

You must show your working.

The gradient of P is k .

$$Q: 6y = -(k + 5)x + 1, \text{ so } y = -\frac{k + 5}{6}x + \frac{1}{6}$$

The gradient of Q is $-\frac{k + 5}{6}$

$$\text{Perpendicular, so the gradients multiply to } -1: k \times \left(-\frac{k + 5}{6}\right) = -1$$

$$\text{Multiply both sides by } -6: k(k + 5) = 6$$

$$k^2 + 5k - 6 = 0$$

$$(k + 6)(k - 1) = 0$$

$$k = -6 \text{ or } k = 1$$

Check $k = 1$: gradients 1 and -1 ✓ Check $k = -6$: gradients -6 and $\frac{1}{6}$ ✓

Answer: $k = 1$ or $k = -6$

Tip: The minus sign belongs to the whole bracket: the gradient of Q is $-\frac{k+5}{6}$, not $\frac{-k+5}{6}$. Keep the bracket until you expand it.

Marks: 1 mark for the gradient of Q . 1 mark for an equation using product = -1 . 1 mark for $k^2 + 5k - 6 = 0$. 1 mark for both values.
