

Equation of a Circle

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A circle has equation $(x + 6)^2 + (y - 1)^2 = 12$

(a) Write down the centre of the circle.

A circle with centre (a, b) and radius r has equation $(x - a)^2 + (y - b)^2 = r^2$.

$$(x + 6)^2 = (x - (-6))^2, \text{ so } a = -6$$

$$(y - 1)^2, \text{ so } b = 1$$

Answer: $(-6, 1)$

Tip: The signs change: $(x + 6)^2$ means the x -coordinate of the centre is -6 , not 6 .

(b) Work out the radius of the circle.

Give your answer as a surd in its simplest form.

The number on the right is r^2 : $r^2 = 12$

$$r = \sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

Answer: $2\sqrt{3}$

Tip: The radius is not 12 (that is r^2) and not 6 . Square root it, then simplify.

2 (1 mark)

A circle has centre $(0, 0)$

The point $(-6, 3)$ lies on the circle.

Circle the equation of the circle.

Options: $x^2 + y^2 = 9$, $x^2 + y^2 = \sqrt{45}$, $x^2 + y^2 = 45$, $x^2 + y^2 = 81$

The radius is the distance from $(0, 0)$ to $(-6, 3)$.

$$r^2 = (-6)^2 + 3^2 = 36 + 9 = 45$$

The equation is $x^2 + y^2 = r^2$, so $x^2 + y^2 = 45$.

Answer: $x^2 + y^2 = 45$

Tip: You do not need to find r : the right-hand side is r^2 , which is $(-6)^2 + 3^2$.

3 (2 marks)

A circle has centre $(2, -3)$ and area 49π

Work out the equation of the circle.

$$\text{Area} = \pi r^2, \text{ so } \pi r^2 = 49\pi$$

$$r^2 = 49 \text{ (so the radius is 7)}$$

Use $(x - a)^2 + (y - b)^2 = r^2$ with $a = 2$ and $b = -3$:

$$(x - 2)^2 + (y - (-3))^2 = 49$$

Answer: $(x - 2)^2 + (y + 3)^2 = 49$

Tip: Watch the signs for the centre: $b = -3$ gives $(y + 3)^2$. The right-hand side is $r^2 = 49$, not 7.

Marks: 1 mark for $r^2 = 49$ (or $r = 7$). 1 mark for the correct equation.

4 (3 marks)

A circle has equation $(x + 1)^2 + (y - 4)^2 = 34$

For each point, tick whether it is inside, on or outside the circle.
You must show your working.

	Inside	On	Outside
$A(2, 9)$		✓	
$B(-4, 1)$	✓		
$C(-6, 0)$			✓

Substitute each point into the left-hand side and compare with 34 (which is r^2).

$A(2, 9)$: $(2 + 1)^2 + (9 - 4)^2 = 9 + 25 = 34$, equal to 34, so A is **on** the circle.

$B(-4, 1)$: $(-4 + 1)^2 + (1 - 4)^2 = 9 + 9 = 18$, less than 34, so B is **inside**.

$C(-6, 0)$: $(-6 + 1)^2 + (0 - 4)^2 = 25 + 16 = 41$, more than 34, so C is **outside**.

(The left-hand side is the square of the distance from the centre $(-1, 4)$ to the point.)

Answer: A on, B inside, C outside

Tip: Bracket negative numbers when you square them: $(-4 + 1)^2 = (-3)^2 = 9$. Compare with $r^2 = 34$, not with r .

Marks: 1 mark for each point with correct working.

5 (3 marks)

The line segment joining $P(-4, 1)$ and $Q(2, 5)$ is a diameter of a circle.
Work out the equation of the circle.

The centre is the midpoint of the diameter: $\left(\frac{-4 + 2}{2}, \frac{1 + 5}{2}\right) = (-1, 3)$

Length of the diameter: $PQ^2 = 6^2 + 4^2 = 52$, so $PQ = \sqrt{52}$

Radius = $\frac{\sqrt{52}}{2}$, so $r^2 = \frac{52}{4} = 13$

(Or: the radius is the distance from the centre to Q : $r^2 = 3^2 + 2^2 = 13$)

$$(x + 1)^2 + (y - 3)^2 = 13$$

Answer: $(x + 1)^2 + (y - 3)^2 = 13$

Tip: PQ is the **diameter**, so halve it to get the radius. The right-hand side is $r^2 = 13$: not 52, not $\sqrt{13}$.

Marks: 1 mark for the centre $(-1, 3)$. 1 mark for $r^2 = 13$ (or $r = \sqrt{13}$). 1 mark for the correct equation.

6 (4 marks)

The point $P(\sqrt{5}, 2)$ lies on the circle $x^2 + y^2 = 9$

Show that the tangent to the circle at P has equation $\sqrt{5}x + 2y = 9$

The centre is $O(0, 0)$. The tangent is perpendicular to the radius OP .

$$\text{Gradient of } OP = \frac{2 - 0}{\sqrt{5} - 0} = \frac{2}{\sqrt{5}}$$

Gradient of the tangent: flip and change the sign, so $-\frac{\sqrt{5}}{2}$

$$\text{Use } y - y_1 = m(x - x_1): y - 2 = -\frac{\sqrt{5}}{2}(x - \sqrt{5})$$

$$\text{Multiply both sides by 2: } 2y - 4 = -\sqrt{5}(x - \sqrt{5})$$

$$\text{Expand: } 2y - 4 = -\sqrt{5}x + 5 \text{ (because } \sqrt{5} \times \sqrt{5} = 5)$$

$$\text{Add } \sqrt{5}x \text{ and 4 to both sides: } \sqrt{5}x + 2y = 9$$

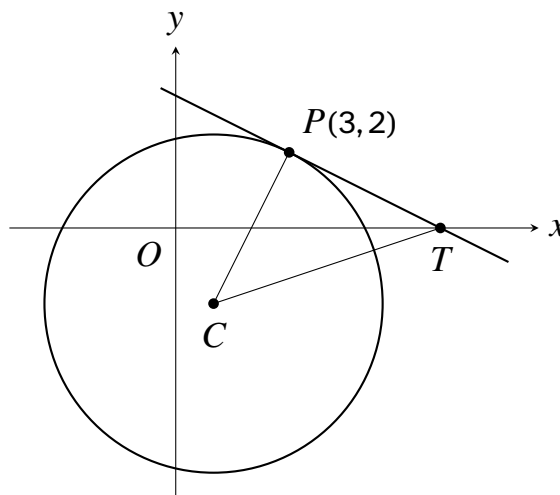
Answer: Tangent gradient $-\frac{\sqrt{5}}{2}$ through P gives $\sqrt{5}x + 2y = 9$

Tip: Most students find the gradients correctly and then lose the last mark rearranging. Clear the fraction first, and remember $-\sqrt{5} \times (-\sqrt{5}) = +5$.

Marks: 1 mark for the gradient of OP . 1 mark for the tangent gradient $-\frac{\sqrt{5}}{2}$. 1 mark for substituting P into $y - y_1 = m(x - x_1)$. 1 mark for rearranging to $\sqrt{5}x + 2y = 9$.

7 (5 marks)

The diagram shows the circle $(x - 1)^2 + (y + 2)^2 = 20$ with centre C .
The point $P(3, 2)$ lies on the circle.



Not drawn accurately

The tangent to the circle at P crosses the x -axis at T .

Work out the area of triangle CPT .

You must show your working.

The centre is $C(1, -2)$.

$$\text{Gradient of the radius } CP = \frac{2 - (-2)}{3 - 1} = \frac{4}{2} = 2$$

The tangent is perpendicular to CP , so its gradient is $-\frac{1}{2}$

$$\text{Tangent: } y - 2 = -\frac{1}{2}(x - 3)$$

$$\text{At } T, y = 0: -2 = -\frac{1}{2}(x - 3), \text{ so } x - 3 = 4 \text{ and } x = 7. \text{ } T \text{ is } (7, 0)$$

Angle $CPT = 90^\circ$ (tangent and radius), so CP and PT are the base and the height.

$$CP = \sqrt{20} \text{ (the radius)}$$

$$PT = \sqrt{(7 - 3)^2 + (0 - 2)^2} = \sqrt{20}$$

$$\text{Area} = \frac{1}{2} \times \sqrt{20} \times \sqrt{20} = \frac{1}{2} \times 20 = 10$$

Answer: 10 units²

Tip: The radius goes from the centre $C(1, -2)$ to P : don't use the origin. The right angle is at P , so use CP and PT in $\frac{1}{2} \times \text{base} \times \text{height}$.

Marks: 1 mark for the gradient of CP . 1 mark for the equation of the tangent. 1 mark for $T(7, 0)$. 1 mark for the lengths CP and PT . 1 mark for 10.

8 (5 marks)

$A(-1, 3)$ and $B(6, 4)$ are two points on a circle whose centre lies on the x -axis. Work out the equation of the circle.

The centre is on the x -axis, so call it $(c, 0)$.

The centre is the same distance from A and from B (both are radii):

$$(c + 1)^2 + (0 - 3)^2 = (c - 6)^2 + (0 - 4)^2$$

$$c^2 + 2c + 1 + 9 = c^2 - 12c + 36 + 16$$

$$2c + 10 = -12c + 52$$

$$14c = 42, \text{ so } c = 3. \text{ The centre is } (3, 0)$$

$$r^2 = (3 + 1)^2 + (0 - 3)^2 = 16 + 9 = 25$$

$$\text{Check with } B: (6 - 3)^2 + 4^2 = 9 + 16 = 25 \quad \checkmark$$

$$\text{The equation is } (x - 3)^2 + y^2 = 25$$

(Another way: the perpendicular bisector of the chord AB passes through the centre. It is $y = -7x + 21$, which meets the x -axis at $(3, 0)$.)

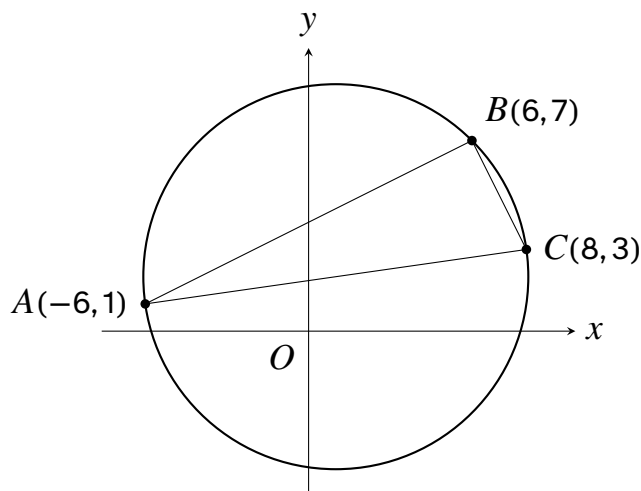
Answer: $(x - 3)^2 + y^2 = 25$

Tip: Expand each squared bracket fully: $(c - 6)^2 = c^2 - 12c + 36$, not $c^2 + 36$. The c^2 terms cancel, leaving a linear equation.

Marks: 1 mark for using a centre $(c, 0)$. 1 mark for an equation for c . 1 mark for $c = 3$. 1 mark for $r^2 = 25$. 1 mark for the correct equation.

9 (5 marks)

$A(-6, 1)$, $B(6, 7)$ and $C(8, 3)$ are on the circumference of a circle.



Not drawn accurately

(a) Show that angle $ABC = 90^\circ$

$$\text{Gradient of } AB = \frac{7-1}{6-(-6)} = \frac{6}{12} = \frac{1}{2}$$

$$\text{Gradient of } BC = \frac{3-7}{8-6} = \frac{-4}{2} = -2$$

$\frac{1}{2} \times (-2) = -1$, so AB is perpendicular to BC and angle $ABC = 90^\circ$.

Answer: The gradients $\frac{1}{2}$ and -2 multiply to -1 , so angle $ABC = 90^\circ$

Marks: 1 mark for both gradients. 1 mark for showing the product is -1 with a conclusion.

(b) Work out the equation of the circle.

Angle $ABC = 90^\circ$, so AC is a diameter (the angle in a semicircle is 90°).

$$\text{Centre} = \text{midpoint of } AC = \left(\frac{-6+8}{2}, \frac{1+3}{2} \right) = (1, 2)$$

$$r^2 = (8-1)^2 + (3-2)^2 = 49 + 1 = 50$$

$$\text{Check with } B: (6-1)^2 + (7-2)^2 = 25 + 25 = 50 \quad \checkmark$$

The equation is $(x - 1)^2 + (y - 2)^2 = 50$

Answer: $(x - 1)^2 + (y - 2)^2 = 50$

Tip: The right angle is at B , so the diameter is AC , the side opposite B . Its midpoint is the centre.

Marks: 1 mark for the centre $(1, 2)$. 1 mark for $r^2 = 50$. 1 mark for the correct equation.
