

# Coordinate Geometry Problems

## Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

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Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

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**1** (3 marks)

*A* is the point  $(-5, 2)$  and *B* is the point  $(3, -6)$

(a) Work out the coordinates of the midpoint of *AB*.

Add the *x*-coordinates and halve:  $\frac{-5 + 3}{2} = \frac{-2}{2} = -1$

Add the *y*-coordinates and halve:  $\frac{2 + (-6)}{2} = \frac{-4}{2} = -2$

**Answer:**  $(-1, -2)$

**Tip:** Midpoint = (average of the *x*'s, average of the *y*'s). Add, then halve: don't halve the differences.

(b) Work out the length of *AB*.

Give your answer in the form  $a\sqrt{b}$  where *a* and *b* are integers.

Change in *x*:  $3 - (-5) = 8$       Change in *y*:  $-6 - 2 = -8$

Pythagoras:  $AB^2 = 8^2 + (-8)^2 = 64 + 64 = 128$

$AB = \sqrt{128}$

The biggest square factor of 128 is 64:  $\sqrt{128} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$

**Answer:**  $8\sqrt{2}$

**Tip:** Look for the **biggest** square factor when you simplify:  $\sqrt{128} = \sqrt{4} \times \sqrt{32} = 2\sqrt{32}$  is not fully simplified.

*Marks:* 1 mark for  $\sqrt{128}$ . 1 mark for  $8\sqrt{2}$ .

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**2** (2 marks)

$P$  is the point  $(-3, 7)$  and  $Q$  is the point  $(9, -1)$

$R$  lies on the line segment  $PQ$  so that  $PR:RQ = 3:1$

Work out the coordinates of  $R$ .

$PR:RQ = 3:1$  means  $PQ$  is split into 4 equal parts, and  $R$  is  $\frac{3}{4}$  of the way from  $P$  to  $Q$ .

From  $P$  to  $Q$ :  $x$  changes by  $9 - (-3) = 12$  and  $y$  changes by  $-1 - 7 = -8$

$\frac{3}{4}$  of these:  $\frac{3}{4} \times 12 = 9$  and  $\frac{3}{4} \times (-8) = -6$

$$R = (-3 + 9, 7 - 6) = (6, 1)$$

**Answer:** (6, 1)

**Tip:** A ratio of 3:1 has 4 parts, so  $R$  is  $\frac{3}{4}$  of the way along, not  $\frac{1}{3}$ . Start from  $P$ , the end named first in the ratio.

*Marks:* 1 mark for  $\frac{3}{4}$  of the change in  $x$  or  $y$  (or 9 or  $-6$ ). 1 mark for (6, 1).

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**3** (3 marks)

The point  $P$  lies on the positive  $x$ -axis.

The distance from  $P$  to the point  $A(4, 6)$  is 10

Work out the coordinates of  $P$ .

$P$  is on the  $x$ -axis, so  $P = (p, 0)$ .

Use Pythagoras:  $(p - 4)^2 + (0 - 6)^2 = 10^2$

$$(p - 4)^2 + 36 = 100$$

$$(p - 4)^2 = 64$$

$$p - 4 = 8 \text{ or } p - 4 = -8$$

$$p = 12 \text{ or } p = -4$$

$P$  is on the **positive**  $x$ -axis, so  $p = 12$ .

**Answer:** (12, 0)

**Tip:** Square rooting gives two values,  $\pm 8$ . Then use the information in the question to choose:  $(-4, 0)$  is on the negative  $x$ -axis.

*Marks:* 1 mark for  $(p - 4)^2 + 6^2 = 10^2$ . 1 mark for  $p = 12$ . 1 mark for (12, 0) with  $p = -4$  rejected.

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**4** (3 marks)

$A$  is the point  $(-4, -1)$ ,  $B$  is the point  $(2, 3)$  and  $C$  is the point  $(11, 9)$   
Show that  $A$ ,  $B$  and  $C$  lie on a straight line.

$$\text{Gradient of } AB = \frac{3 - (-1)}{2 - (-4)} = \frac{4}{6} = \frac{2}{3}$$

$$\text{Gradient of } BC = \frac{9 - 3}{11 - 2} = \frac{6}{9} = \frac{2}{3}$$

$AB$  and  $BC$  have the same gradient, so they are parallel.

They also share the point  $B$ , so  $A$ ,  $B$  and  $C$  lie on one straight line.

**Answer:**  $AB$  and  $BC$  both have gradient  $\frac{2}{3}$  and share point  $B$ , so  $A$ ,  $B$  and  $C$  are on a straight line

**Tip:** Equal gradients on their own only show that  $AB$  and  $BC$  are parallel. Say that they share the point  $B$  as well.

*Marks:* 1 mark for one correct gradient. 1 mark for both gradients. 1 mark for a conclusion that mentions the same gradient and the common point  $B$ .

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**5** (5 marks)

$A$  is the point  $(-3, 2)$ ,  $B$  is the point  $(1, 5)$  and  $C$  is the point  $(4, 1)$

(a) Show that triangle  $ABC$  is right-angled.

$$\text{Gradient of } AB = \frac{5 - 2}{1 - (-3)} = \frac{3}{4}$$

$$\text{Gradient of } BC = \frac{1 - 5}{4 - 1} = -\frac{4}{3}$$

$$\frac{3}{4} \times \left(-\frac{4}{3}\right) = -1, \text{ so } AB \text{ is perpendicular to } BC.$$

So angle  $ABC = 90^\circ$  and the triangle is right-angled.

(Another way:  $AB^2 = 4^2 + 3^2 = 25$ ,  $BC^2 = 3^2 + 4^2 = 25$ ,  $AC^2 = 7^2 + 1^2 = 50$ , and  $25 + 25 = 50$ .)

**Answer:** The gradients of  $AB$  and  $BC$  multiply to  $-1$ , so angle  $ABC = 90^\circ$

**Tip:** Check which pair of sides meets at a right angle:  $AB$  and  $BC$  meet at  $B$ . Finish with a statement, such as “so angle  $ABC = 90^\circ$ ”.

*Marks:* 1 mark for one gradient (or one length<sup>2</sup>). 1 mark for both gradients and their product  $-1$  (or all three lengths<sup>2</sup>). 1 mark for a conclusion that the right angle is at  $B$ .

(b) Work out the area of triangle  $ABC$ .

The right angle is at  $B$ , so  $AB$  and  $BC$  are the base and the height.

$$AB = \sqrt{4^2 + 3^2} = 5 \quad \text{and} \quad BC = \sqrt{3^2 + (-4)^2} = 5$$

$$\text{Area} = \frac{1}{2} \times 5 \times 5 = 12.5$$

**Answer:** 12.5 units<sup>2</sup>

**Tip:** Use the two sides that meet at the right angle.  $AC$  is the hypotenuse, so it is not the base or the height. Remember the  $\frac{1}{2}$ .

*Marks:* 1 mark for  $AB = 5$  and  $BC = 5$ . 1 mark for 12.5.

**6** (3 marks)

$A$  is the point  $(-5, 6)$  and  $B$  is the point  $(3, 2)$

The point  $M$  lies on the straight line through  $A$  and  $B$ , beyond  $B$ , so that

$$AM:MB = 7:3$$

Work out the coordinates of  $M$ .

Sketch the order of the points:  $A - B - M$

$AM$  is 7 parts and  $MB$  is 3 parts, so  $AB$  is  $7 - 3 = 4$  parts.

From  $A$  to  $B$ :  $x$  changes by  $3 - (-5) = 8$  and  $y$  changes by  $2 - 6 = -4$

One part:  $8 \div 4 = 2$  and  $-4 \div 4 = -1$

From  $B$  to  $M$  is 3 parts:  $x$  changes by 6 and  $y$  changes by  $-3$

$$M = (3 + 6, 2 - 3) = (9, -1)$$

Check: from  $A$  to  $M$  is  $(14, -7)$  and from  $M$  to  $B$  is  $(-6, 3)$ , which are in the ratio 7:3 ✓

**Answer:**  $(9, -1)$

**Tip:** Draw a quick sketch of the line.  $M$  is beyond  $B$ , so  $AB$  is not 7 parts: it is  $AM - MB = 4$  parts.

*Marks:* 1 mark for  $AB$  being 4 parts (or one part =  $(2, -1)$ ). 1 mark for the step from  $B$  to  $M$ ,  $(6, -3)$ , or a correct method for  $M$ . 1 mark for  $(9, -1)$ .

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**7** (4 marks)

$A$  is the point  $(-2, 5)$  and  $B$  is the point  $(6, 1)$

The perpendicular bisector of  $AB$  crosses the  $y$ -axis at the point  $C$ .

Work out the coordinates of  $C$ .

The perpendicular bisector passes through the midpoint of  $AB$  at right angles to  $AB$ .

$$\text{Midpoint of } AB = \left( \frac{-2 + 6}{2}, \frac{5 + 1}{2} \right) = (2, 3)$$

$$\text{Gradient of } AB = \frac{1 - 5}{6 - (-2)} = \frac{-4}{8} = -\frac{1}{2}$$

Perpendicular gradient = 2

$$\text{Perpendicular bisector: } y - 3 = 2(x - 2), \text{ so } y = 2x - 1$$

On the  $y$ -axis,  $x = 0$ :  $y = -1$

Check:  $CA^2 = 2^2 + 6^2 = 40$  and  $CB^2 = 6^2 + 2^2 = 40$ , so  $C$  is the same distance from  $A$  and  $B$  ✓

**Answer:**  $(0, -1)$

**Tip:** The perpendicular bisector goes through the **midpoint** of  $AB$ , not through  $A$  or  $B$ , and its gradient is the negative reciprocal of the gradient of  $AB$ .

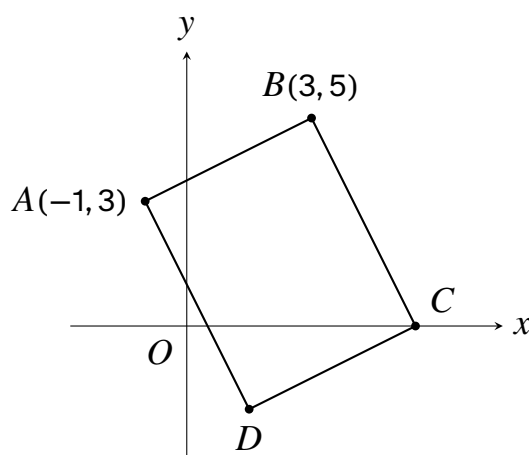
**Marks:** 1 mark for the midpoint  $(2, 3)$ . 1 mark for the perpendicular gradient 2. 1 mark for the equation  $y = 2x - 1$ . 1 mark for  $(0, -1)$ .

**8** (6 marks)

$ABCD$  is a rectangle.

$A$  is the point  $(-1, 3)$  and  $B$  is the point  $(3, 5)$

$C$  lies on the  $x$ -axis.



*Not drawn accurately*

(a) Work out the coordinates of  $C$ .

$$\text{Gradient of } AB = \frac{5 - 3}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

Angle  $ABC = 90^\circ$ , so  $BC$  is perpendicular to  $AB$  and its gradient is  $-2$ .

$C$  is on the  $x$ -axis, so  $C = (c, 0)$ .

$$\text{Gradient of } BC = \frac{0 - 5}{c - 3} = -2$$

$$-5 = -2(c - 3) = -2c + 6$$

$$2c = 11, \text{ so } c = 5.5$$

**Answer:** (5.5, 0)

**Tip:** A rectangle has right angles, so neighbouring sides have perpendicular gradients. Points on the  $x$ -axis have  $y$ -coordinate 0.

*Marks:* 1 mark for the gradient of  $BC$ ,  $-2$ . 1 mark for a correct equation for  $c$  (or the equation of line  $BC$ ). 1 mark for (5.5, 0).

- (b) Work out the coordinates of  $D$ .

$AD$  and  $BC$  are opposite sides of the rectangle, so the move from  $A$  to  $D$  is the same as the move from  $B$  to  $C$ .

From  $B$  to  $C$ :  $x$  changes by  $5.5 - 3 = 2.5$  and  $y$  changes by  $0 - 5 = -5$

$$D = (-1 + 2.5, 3 - 5) = (1.5, -2)$$

**Answer:** (1.5, -2)

- (c) Work out the area of the rectangle.

$$AB = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$BC = \sqrt{2.5^2 + 5^2} = \sqrt{31.25}$$

$$\text{Area} = \sqrt{20} \times \sqrt{31.25} = \sqrt{625} = 25$$

**Answer:** 25 units<sup>2</sup>

**Tip:** Keep the lengths as surds (or the full calculator values) and multiply at the end. Rounding  $\sqrt{20}$  to 4.5 gives a wrong area.

*Marks:* 1 mark for the lengths of  $AB$  and  $BC$ . 1 mark for 25.