

Circle Theorems

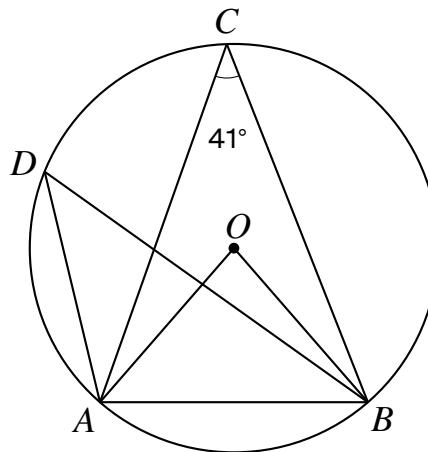
Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

A, B, C and D are points on a circle, centre O .



Not drawn accurately

- (a) Angle $ACB = 41^\circ$
Give a reason why angle ADB is also 41° .

Angles ACB and ADB both stand on the chord AB .

C and D are on the same side of AB , so the angles are in the same segment.

Angles in the same segment are equal.

Answer: Angles in the same segment are equal.

Tip: Use the proper name of the theorem. "They come from the same chord" or "bow-tie" does not get the mark.

- (b) Work out the size of angle OAB .

Give reasons for your answer.

$$\text{Angle } AOB = 2 \times 41 = 82^\circ$$

(The angle at the centre is twice the angle at the circumference.)

$OA = OB$ (radii), so triangle OAB is isosceles.

$$\text{Angle } OAB = (180 - 82) \div 2 = 49^\circ$$

(Base angles of an isosceles triangle are equal; angles in a triangle add up to 180° .)

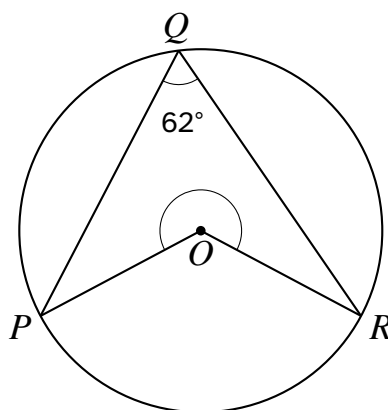
Answer: Angle $OAB = 49^\circ$

Tip: Say **why** triangle OAB is isosceles: OA and OB are both radii.

Marks: 1 mark for angle $AOB = 82^\circ$. 1 mark for 49° with reasons.

2 (1 mark)

P , Q and R are points on a circle, centre O .



Not drawn accurately

$$\text{Angle } PQR = 62^\circ$$

What is the size of the reflex angle POR ?

Circle your answer.

Options: 118° , 124° , 236° , 298°

Angle PQR is at the circumference, on the major arc.

So the angle at the centre on the same arc is angle $POR = 2 \times 62 = 124^\circ$.

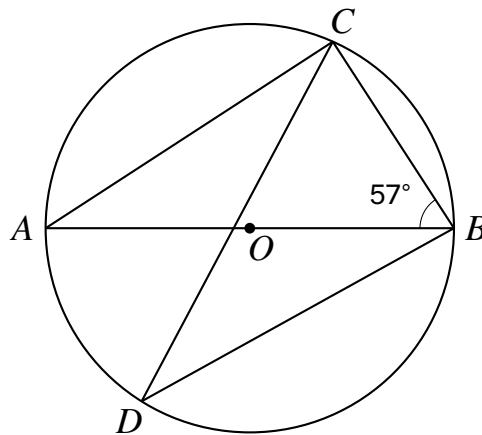
The reflex angle $POR = 360 - 124 = 236^\circ$

Answer: 236°

Tip: A reflex angle is more than 180° . Check which angle at the centre stands on the same arc as the angle at the circumference.

3 (3 marks)

A, B, C and D are points on a circle, centre O .



Not drawn accurately

AB is a diameter of the circle.

Angle $ABC = 57^\circ$

Work out the size of angle BDC .

Give reasons for your answer.

Angle $ACB = 90^\circ$

(The angle in a semicircle is 90° .)

Angle $BAC = 180 - 90 - 57 = 33^\circ$

(Angles in a triangle add up to 180° .)

Angles BDC and BAC both stand on the chord BC , in the same segment.

Angle $BDC = 33^\circ$

(Angles in the same segment are equal.)

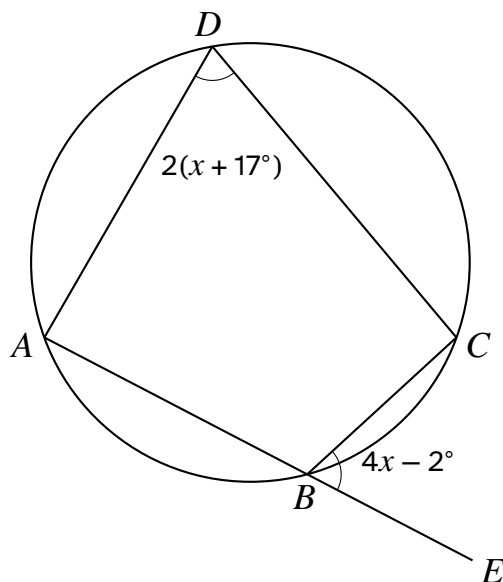
Answer: Angle $BDC = 33^\circ$

Tip: Look for the angle that stands on the same chord as the angle you want. Here that is angle BAC , which you can find from triangle ABC .

Marks: 1 mark for angle $ACB = 90^\circ$ or angle $BAC = 33^\circ$. 1 mark for 33° . 1 mark for all three reasons.

4 (3 marks)

A, B, C and D are points on a circle.



Not drawn accurately

$ABCD$ is a cyclic quadrilateral.

ABE is a straight line.

Angle $ADC = 2(x + 17^\circ)$ Angle $CBE = 4x - 2^\circ$

Work out the size of angle ABC .

$$\text{Angle } ABC = 180 - (4x - 2) = 182 - 4x$$

(Angles on a straight line add up to 180° .)

Angles ABC and ADC are opposite angles of the cyclic quadrilateral, so they add up to 180° .

$$182 - 4x + 2x + 34 = 180$$

$$216 - 2x = 180$$

$$2x = 36, \text{ so } x = 18$$

$$\text{Angle } ABC = 182 - 4 \times 18 = 110^\circ$$

(Check: angle $ADC = 2(18 + 17) = 70^\circ$ and $110 + 70 = 180$.)

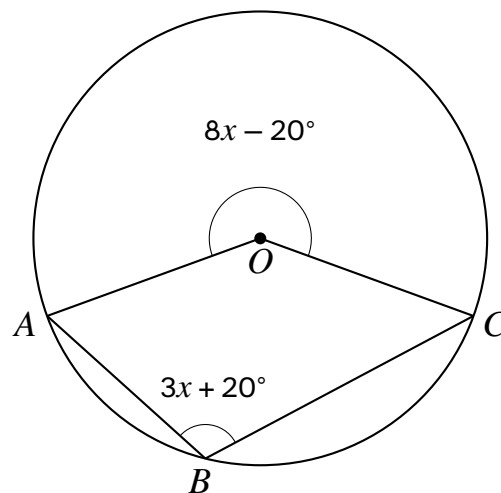
Answer: Angle $ABC = 110^\circ$

Tip: $2(x + 17)$ is $2x + 34$, not $2x + 17$. Multiply every term in the bracket. Another way: the exterior angle of a cyclic quadrilateral equals the interior opposite angle, so $4x - 2 = 2(x + 17)$.

Marks: 1 mark for a correct equation in x . 1 mark for $x = 18$. 1 mark for 110° .

5 (5 marks)

A, B and C are points on a circle, centre O .



Not drawn accurately

(a) Work out the value of x .

B is on the minor arc AC , so angle ABC stands on the major arc.

The angle at the centre on the major arc is the reflex angle AOC .

Reflex angle $AOC = 2 \times \text{angle } ABC$

(The angle at the centre is twice the angle at the circumference.)

$$8x - 20 = 2(3x + 20)$$

$$8x - 20 = 6x + 40$$

$$2x = 60$$

$$x = 30$$

Answer: $x = 30$

Tip: Here the angle at the centre that goes with angle ABC is the **reflex** angle. Doubling $3x + 20$ and setting it equal to $360 - (8x - 20)$ is a common mistake.

Marks: 1 mark for $8x - 20 = 2(3x + 20)$. 1 mark for collecting terms correctly. 1 mark for $x = 30$.

(b) Angle $OCB = 48^\circ$

Work out the size of angle OAB .

Give reasons for your answer.

$$\text{Angle } ABC = 3 \times 30 + 20 = 110^\circ$$

Join OB .

$$OB = OC \text{ (radii), so angle } OBC = 48^\circ$$

(Base angles of an isosceles triangle are equal.)

$$\text{Angle } OBA = 110 - 48 = 62^\circ$$

$$OA = OB \text{ (radii), so angle } OAB = \text{angle } OBA = 62^\circ$$

(Base angles of an isosceles triangle are equal.)

Answer: Angle $OAB = 62^\circ$

Tip: Draw in the radius OB . It splits angle ABC into angle OBA and angle OBC . Use the value of x from part (a) to find angle ABC first.

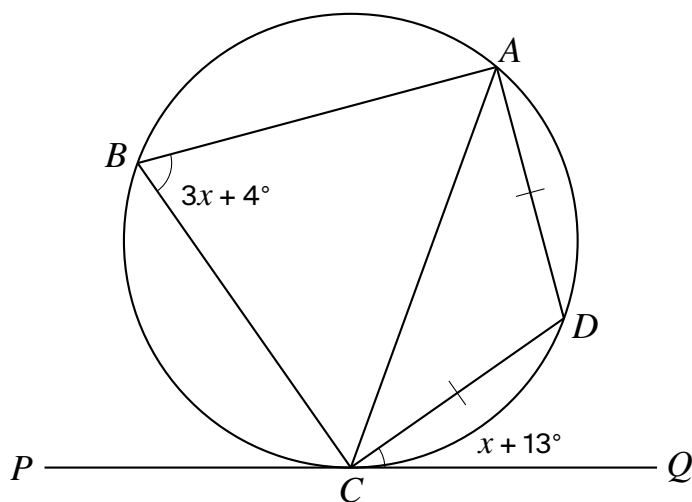
Marks: 1 mark for angle $OBA = 62^\circ$. 1 mark for angle $OAB = 62^\circ$ with reasons.

6 (4 marks)

A, B, C and D are points on a circle.

PCQ is the tangent to the circle at C .

$AD = CD$



Not drawn accurately

Work out the value of x .

Angle $DAC = \text{angle } DCQ = x + 13$

(Alternate segment theorem.)

$AD = CD$, so triangle ACD is isosceles and angle $DCA = \text{angle } DAC = x + 13$

(Base angles of an isosceles triangle are equal.)

Angle $ADC = 180 - 2(x + 13) = 154 - 2x$

(Angles in a triangle add up to 180° .)

Angles ABC and ADC are opposite angles of the cyclic quadrilateral $ABCD$, so they add up to 180° .

$$(3x + 4) + (154 - 2x) = 180$$

$$x + 158 = 180$$

$$x = 22$$

(Check: angle $ABC = 70^\circ$ and angle $ADC = 110^\circ$; $70 + 110 = 180$.)

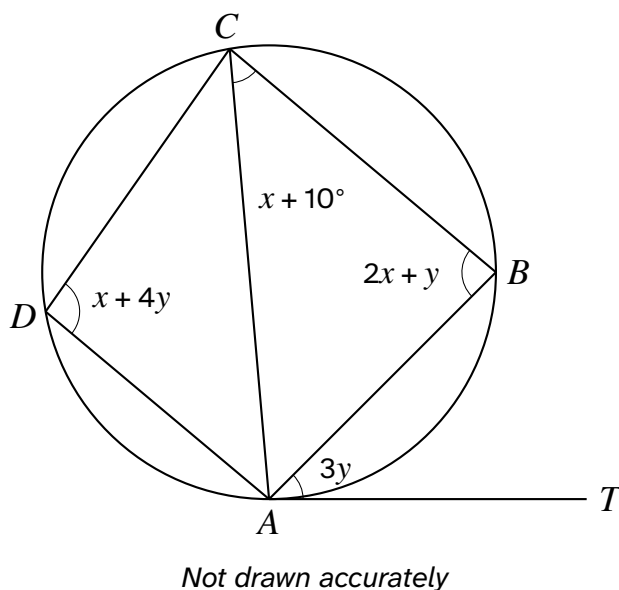
Answer: $x = 22$

Tip: Angle DCQ is between the tangent and the chord CD , so by the alternate segment theorem it equals the angle that CD makes at A , angle DAC . Use brackets: $180 - 2(x + 13)$ is $154 - 2x$. Another route: angle $ACQ = 2(x + 13)$, and this equals angle ABC (alternate segment theorem).

Marks: 1 mark for angle $DAC = x + 13$ (or angle $DCA = x + 13$). 1 mark for angle $ADC = 154 - 2x$. 1 mark for a correct equation. 1 mark for $x = 22$.

7 (4 marks)

A, B, C and D are points on a circle.
 AT is the tangent to the circle at A .



Work out the value of x and the value of y .

Angle $ACB =$ angle TAB (alternate segment theorem)

$$x + 10 = 3y \quad (1)$$

Angles ABC and ADC are opposite angles of the cyclic quadrilateral, so they add up to 180° .

$$(2x + y) + (x + 4y) = 180$$

$$3x + 5y = 180 \quad (2)$$

From (1): $x = 3y - 10$

Substitute into (2): $3(3y - 10) + 5y = 180$

$14y - 30 = 180$, so $14y = 210$ and $y = 15$

$x = 3 \times 15 - 10 = 35$

(Check: angle $ABC = 85^\circ$ and angle $ADC = 95^\circ$, which add up to 180° .)

Answer: $x = 35$, $y = 15$

Tip: You need two equations, one from each theorem. Check which pair of angles each theorem links: the opposite angles here are at B and D , not at A and C .

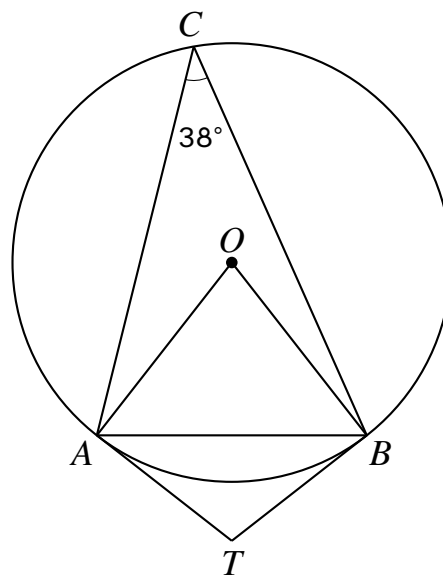
Marks: 1 mark for $x + 10 = 3y$. 1 mark for $3x + 5y = 180$. 1 mark for eliminating x or y correctly. 1 mark for both values.

8 (6 marks)

A , B and C are points on a circle, centre O , radius 6 cm.

TA and TB are tangents to the circle.

Angle $ACB = 38^\circ$



Not drawn accurately

- (a) Work out the size of angle ATB .
Give reasons for your answer.

$$\text{Angle } AOB = 2 \times 38 = 76^\circ$$

(The angle at the centre is twice the angle at the circumference.)

$$\text{Angle } OAT = \text{angle } OBT = 90^\circ$$

(A tangent is perpendicular to the radius.)

$$\text{Angle } ATB = 360 - 90 - 90 - 76 = 104^\circ$$

(Angles in a quadrilateral add up to 360° .)

Answer: Angle $ATB = 104^\circ$

Marks: 1 mark for angle $AOB = 76^\circ$. 1 mark for 104° with reasons.

- (b) Work out the perimeter of triangle ATB .

Give your answer to 3 significant figures.

$TA = TB$ (tangents from a point are equal), so $OATB$ is a kite and OT bisects angle AOB .

In right-angled triangle OAT : angle $AOT = 76 \div 2 = 38^\circ$ and $OA = 6$ cm

$$TA = 6 \tan 38^\circ = 4.6877 \dots \text{ cm}$$

The perpendicular from O to the chord AB bisects AB . Call its foot M .

In right-angled triangle OAM : $AM = 6 \sin 38^\circ = 3.6939 \dots$ cm

$$AB = 2 \times 3.6939 \dots = 7.3878 \dots \text{ cm}$$

$$\text{Perimeter} = 4.6877 \dots + 4.6877 \dots + 7.3878 \dots = 16.763 \dots$$

$$= 16.8 \text{ cm (to 3 s.f.)}$$

Answer: 16.8 cm

Tip: Keep full calculator values until the end; rounding TA and AB first can give the wrong final answer. You could also use the cosine rule for AB : $AB^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \cos 76^\circ$.

Marks: 1 mark for TA or $TB = 4.69$. 1 mark for $AB = 7.39$. 1 mark for adding all three sides. 1 mark for 16.8.