

Area, Perimeter and Volume with Algebra

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

A square has sides of length x cm.

A circle has the same perimeter as the square.

(a) Work out the radius of the circle in terms of x and π .

$$\text{Perimeter of the square} = 4x$$

$$\text{Circumference of the circle} = 2\pi r$$

$$2\pi r = 4x, \text{ so } r = \frac{4x}{2\pi} = \frac{2x}{\pi}$$

Answer: $r = \frac{2x}{\pi}$

(b) Show that area of the circle : area of the square = $4 : \pi$

$$\text{Area of the circle} = \pi r^2 = \pi \times \left(\frac{2x}{\pi}\right)^2 = \pi \times \frac{4x^2}{\pi^2} = \frac{4x^2}{\pi}$$

$$\text{Area of the square} = x^2$$

$$\text{Ratio} = \frac{4x^2}{\pi} : x^2$$

$$\text{Divide both parts by } x^2: \frac{4}{\pi} : 1$$

Multiply both parts by π : $4:\pi$

Answer: $\frac{4x^2}{\pi}:x^2 = 4:\pi$

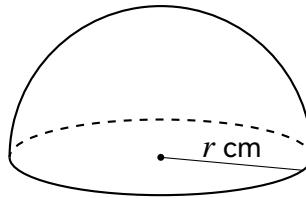
Tip: Square the whole fraction: $\left(\frac{2x}{\pi}\right)^2 = \frac{4x^2}{\pi^2}$. Then one π cancels.

Marks: 1 mark for the area of the circle as $\frac{4x^2}{\pi}$. 1 mark for completing the ratio to $4:\pi$.

2 (3 marks)

A solid hemisphere has radius r cm.

Surface area of a sphere = $4\pi r^2$



Not drawn accurately

Its volume, in cm^3 , and its total surface area, in cm^2 , have the same value.
Work out the value of r .

A hemisphere is half a sphere: volume = $\frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$

Curved surface = $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$

Flat circular face = πr^2

Total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

Divide both sides by πr^2 (r is not 0): $\frac{2}{3}r = 3$

$$r = 3 \times \frac{3}{2} = 4.5$$

Answer: $r = 4.5$

Tip: A **solid** hemisphere has a flat circular face as well as its curved surface: the total surface area is $2\pi r^2 + \pi r^2$, not $2\pi r^2$.

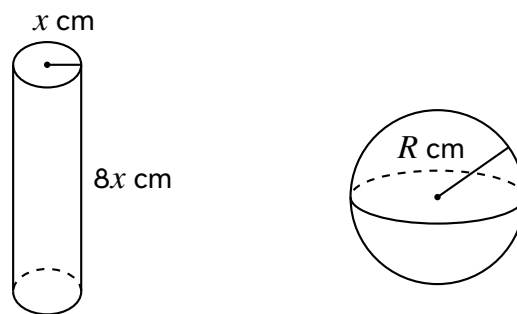
Marks: 1 mark for the total surface area $3\pi r^2$ (or $2\pi r^2 + \pi r^2$). 1 mark for $\frac{2}{3}\pi r^3 = 3\pi r^2$. 1 mark for $r = 4.5$.

3 (3 marks)

A cylinder has radius x cm and height $8x$ cm.

A sphere has radius R cm.

The cylinder and the sphere have the same volume.



Not drawn accurately

Work out R in terms of x .

Give your answer to 3 significant figures.

$$\text{Volume of the cylinder} = \pi r^2 h = \pi \times x^2 \times 8x = 8\pi x^3$$

$$\text{Volume of the sphere} = \frac{4}{3}\pi R^3$$

$$\frac{4}{3}\pi R^3 = 8\pi x^3$$

$$\text{Divide by } \pi \text{ and multiply by } \frac{3}{4}: R^3 = 6x^3$$

$$\text{Cube root: } R = \sqrt[3]{6}x = 1.817\dots x$$

To 3 significant figures: $R = 1.82x$

Answer: $R = 1.82x$ (exactly, $R = \sqrt[3]{6}x$)

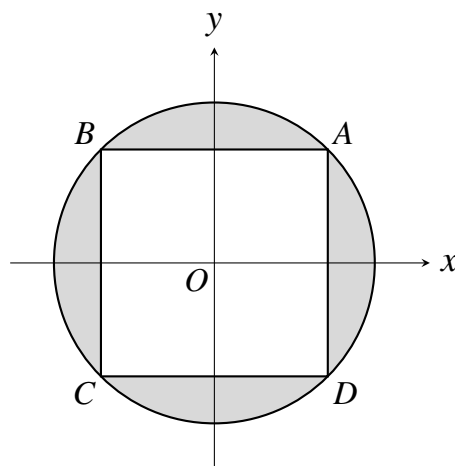
Tip: $x^2 \times 8x = 8x^3$: add the powers. The cube root of $6x^3$ is $\sqrt[3]{6} \times x$, because $\sqrt[3]{x^3} = x$.

Marks: 1 mark for $8\pi x^3$ (the volume of the cylinder). 1 mark for $R^3 = 6x^3$. 1 mark for $1.82x$.

4 (3 marks)

The points $A(5, 5)$, $B(-5, 5)$, $C(-5, -5)$ and $D(5, -5)$ lie on a circle with centre O , the origin.

$ABCD$ is a square.



Not drawn accurately

Work out the area of the shaded region.

Give your answer in the form $a\pi - b$, where a and b are integers.

The radius is the distance OA : $r^2 = 5^2 + 5^2 = 50$

Area of the circle = $\pi r^2 = 50\pi$

The square has side $5 - (-5) = 10$, so its area is $10^2 = 100$

$$\text{Shaded area} = 50\pi - 100$$

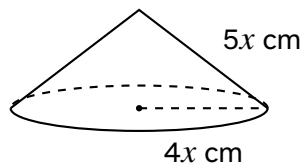
Answer: $50\pi - 100$

Tip: You need r^2 for the area, so there is no need to find $r = \sqrt{50}$ itself. The equation of the circle is $x^2 + y^2 = 50$, which gives r^2 directly.

Marks: 1 mark for $r^2 = 50$ (or $r = \sqrt{50}$). 1 mark for 50π or 100. 1 mark for $50\pi - 100$.

5 (4 marks)

A solid cone has base radius $4x$ cm and slant height $5x$ cm.
The total surface area of the cone is 81π cm².



Not drawn accurately

(a) Work out the value of x .

$$\text{Curved surface area} = \pi r l = \pi \times 4x \times 5x = 20\pi x^2$$

$$\text{Area of the circular base} = \pi r^2 = \pi (4x)^2 = 16\pi x^2$$

$$\text{Total surface area} = 20\pi x^2 + 16\pi x^2 = 36\pi x^2$$

$$36\pi x^2 = 81\pi, \text{ so } x^2 = \frac{81}{36} = 2.25$$

$$x = 1.5 \text{ (a length cannot be negative)}$$

Answer: $x = 1.5$

Tip: The **total** surface area includes the circular base, πr^2 , as well as the curved surface $\pi r l$. Leaving out the base is the most common mistake.

Marks: 1 mark for $20\pi x^2 + 16\pi x^2$ (or $36\pi x^2$). 1 mark for $x = 1.5$.

(b) Work out the volume of the cone.
Give your answer in terms of π .

$$\text{With } x = 1.5: \text{ radius} = 6 \text{ cm and slant height} = 7.5 \text{ cm.}$$

Perpendicular height: $h = \sqrt{7.5^2 - 6^2} = \sqrt{20.25} = 4.5 \text{ cm}$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \pi \times 36 \times 4.5$$

$$= 54\pi$$

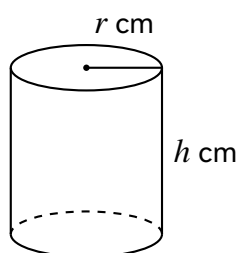
Answer: $54\pi \text{ cm}^3$

Tip: The volume needs the **perpendicular** height, not the slant height. Use Pythagoras first: the sides $4x$, $3x$ and $5x$ make a 3, 4, 5 triangle, so $h = 3x$.

Marks: 1 mark for the height 4.5 (using Pythagoras). 1 mark for 54π .

6 (3 marks)

A closed cylinder has radius $r \text{ cm}$ and height $h \text{ cm}$.
The total surface area of the cylinder is $600\pi \text{ cm}^2$.
The volume of the cylinder is $V \text{ cm}^3$.



Not drawn accurately

Show that $V = 300\pi r - \pi r^3$

Each circular end has area πr^2 , and the curved surface has area $2\pi r \times h = 2\pi rh$

Total surface area: $2\pi r^2 + 2\pi rh = 600\pi$

Divide by 2π : $r^2 + rh = 300$

So $rh = 300 - r^2$, which gives $h = \frac{300 - r^2}{r}$

$$V = \pi r^2 h = \pi r^2 \times \frac{300 - r^2}{r} = \pi r(300 - r^2)$$

$$V = 300\pi r - \pi r^3$$

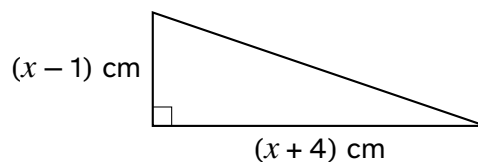
Answer: $2\pi r^2 + 2\pi rh = 600\pi$ gives $h = \frac{300 - r^2}{r}$, so $V = \pi r^2 h = 300\pi r - \pi r^3$

Tip: A closed cylinder has **two** circular ends. The curved surface opens out into a rectangle $2\pi r$ long and h high.

Marks: 1 mark for $2\pi r^2 + 2\pi rh = 600\pi$. 1 mark for h (or rh) in terms of r . 1 mark for substituting into $\pi r^2 h$ and simplifying to the given answer.

7 (4 marks)

The diagram shows a right-angled triangle.



Not drawn accurately

The area of the triangle is less than 7 cm^2 .

Work out the range of possible values of x .

You must show your working.

$$\text{Area} = \frac{1}{2}(x + 4)(x - 1) < 7$$

$$\text{Multiply by 2: } (x + 4)(x - 1) < 14$$

$$\text{Expand: } x^2 + 3x - 4 < 14, \text{ so } x^2 + 3x - 18 < 0$$

$$\text{Factorise: } (x + 6)(x - 3) < 0$$

The graph of $(x + 6)(x - 3)$ is a U-shape crossing the x -axis at -6 and 3 , so $-6 < x < 3$

But the side $(x - 1)$ must be a positive length: $x - 1 > 0$, so $x > 1$

Both conditions together: $1 < x < 3$

Answer: $1 < x < 3$

Tip: Lengths must be positive. Use $x - 1 > 0$ for the lower limit: many students give $-6 < x < 3$ and lose the last mark.

Marks: 1 mark for $\frac{1}{2}(x + 4)(x - 1) < 7$. 1 mark for $x^2 + 3x - 18 < 0$. 1 mark for the critical values -6 and 3 (or $-6 < x < 3$). 1 mark for $1 < x < 3$.
