

Algebraic Fractions

Solutions

For AQA Level 2 Certificate in Further Mathematics (8365)

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

Which of these is equal to $\frac{a}{3} + \frac{2}{b}$?

Circle your answer.

Options: $\frac{a+2}{3+b}$, $\frac{ab+6}{3b}$, $\frac{a+2}{3b}$, $\frac{ab+2}{3b}$

The common denominator is $3b$.

$$\frac{a}{3} = \frac{ab}{3b} \quad \text{and} \quad \frac{2}{b} = \frac{6}{3b}$$

Add the numerators: $\frac{ab+6}{3b}$

Answer: $\frac{ab+6}{3b}$

Tip: Never add the tops and the bottoms separately. Write both fractions over the common denominator first, and multiply **both** numerators.

2 (4 marks)

Write each expression as a single fraction in its simplest form.

(a) $\frac{7}{10x} - \frac{2}{15x}$

The lowest common multiple of $10x$ and $15x$ is $30x$.

$$\frac{7}{10x} = \frac{21}{30x} \quad (\text{multiply top and bottom by 3})$$

$$\frac{2}{15x} = \frac{4}{30x} \quad (\text{multiply top and bottom by 2})$$

Subtract the numerators: $21 - 4 = 17$

Answer: $\frac{17}{30x}$

Tip: Using $10x \times 15x = 150x^2$ as the denominator works, but you must then simplify $\frac{85x}{150x^2}$ fully. The lowest common denominator saves that step.

Marks: 1 mark for writing both fractions over a common denominator, e.g. $\frac{105x - 20x}{150x^2}$. 1 mark for $\frac{17}{30x}$.

(b) $\frac{5}{6x} - \frac{3}{4x^2}$

The lowest common denominator of $6x$ and $4x^2$ is $12x^2$.

$$\frac{5}{6x} = \frac{10x}{12x^2} \quad (\text{multiply top and bottom by } 2x)$$

$$\frac{3}{4x^2} = \frac{9}{12x^2} \quad (\text{multiply top and bottom by } 3)$$

Subtract the numerators.

Answer: $\frac{10x - 9}{12x^2}$

Tip: Whatever you multiply the bottom by, multiply the top by the same: $6x \times 2x = 12x^2$, so 5 becomes $10x$.

Marks: 1 mark for the common denominator $12x^2$ with one numerator correct. 1 mark for the correct answer.

3 (3 marks)

Simplify fully $\frac{x^3 - 6x^2 + 9x}{x^2 - 9}$

Factorise the numerator. First take out x : $x(x^2 - 6x + 9)$

$$x^2 - 6x + 9 = (x - 3)(x - 3), \text{ so the numerator is } x(x - 3)^2.$$

Factorise the denominator (difference of two squares): $(x + 3)(x - 3)$

$$\frac{x(x-3)(x-3)}{(x+3)(x-3)}$$

Cancel the common factor $(x-3)$.

Answer: $\frac{x(x-3)}{x+3}$

Tip: You can only cancel whole factors (brackets), never single terms. Cancelling the x^2 on top with the x^2 underneath is wrong.

Marks: 1 mark for $x(x-3)^2$. 1 mark for $(x+3)(x-3)$. 1 mark for the correct answer.

4 (5 marks)

Simplify fully

(a) $\frac{6x^2 + 7x - 3}{4x^2 - 9}$

Factorise the numerator: $6 \times (-3) = -18$, and 9 and -2 multiply to -18 and add to 7.

$$6x^2 + 9x - 2x - 3 = 3x(2x + 3) - 1(2x + 3) = (3x - 1)(2x + 3)$$

Factorise the denominator (difference of two squares): $4x^2 - 9 = (2x + 3)(2x - 3)$

$$\frac{(3x - 1)(2x + 3)}{(2x + 3)(2x - 3)}$$

Cancel the common factor $(2x + 3)$.

Answer: $\frac{3x - 1}{2x - 3}$

Tip: Factorise directly, and expand your brackets to check the signs. Working out the roots and trying to work backwards nearly always goes wrong.

Marks: 1 mark for $(3x - 1)(2x + 3)$. 1 mark for $(2x + 3)(2x - 3)$. 1 mark for the correct answer.

(b) $\frac{10 - 2x}{x^2 - 3x - 10}$

Factorise the denominator: $x^2 - 3x - 10 = (x - 5)(x + 2)$

Factorise the numerator so that it contains $(x - 5)$: $10 - 2x = -2(x - 5)$

$$\frac{-2(x-5)}{(x-5)(x+2)}$$

Cancel $(x-5)$.

Answer: $-\frac{2}{x+2}$ (or $\frac{-2}{x+2}$)

Tip: $10-2x$ and $x-5$ look different, but $10-2x = -2(x-5)$. Don't lose the minus sign when you cancel.

Marks: 1 mark for $(x-5)(x+2)$ and $-2(x-5)$ (or $2(5-x)$). 1 mark for the correct answer.

5 (3 marks)

Simplify fully $\frac{12a^2b - 8ab^2}{9a^2 - 4b^2}$

Factorise the numerator: the highest common factor is $4ab$.

$$12a^2b - 8ab^2 = 4ab(3a - 2b)$$

Factorise the denominator (difference of two squares): $9a^2 - 4b^2 = (3a + 2b)(3a - 2b)$

$$\frac{4ab(3a - 2b)}{(3a + 2b)(3a - 2b)}$$

Cancel the common factor $(3a - 2b)$.

Answer: $\frac{4ab}{3a + 2b}$

Marks: 1 mark for $4ab(3a - 2b)$. 1 mark for $(3a + 2b)(3a - 2b)$. 1 mark for the correct answer.

6 (3 marks)

Write as a single fraction in its simplest form

$$\frac{4}{x-3} - \frac{3}{2x+1}$$

The common denominator is $(x-3)(2x+1)$.

$$\frac{4}{x-3} = \frac{4(2x+1)}{(x-3)(2x+1)} \quad \text{and} \quad \frac{3}{2x+1} = \frac{3(x-3)}{(x-3)(2x+1)}$$

Subtract the numerators, with brackets: $4(2x + 1) - 3(x - 3)$

$$= 8x + 4 - 3x + 9$$

$$= 5x + 13$$

Answer: $\frac{5x + 13}{(x - 3)(2x + 1)}$

Tip: $-3(x - 3) = -3x + 9$, not $-3x - 9$. Keep the brackets when you subtract, and multiply **every** term inside.

Marks: 1 mark for the common denominator $(x - 3)(2x + 1)$. 1 mark for the numerator $4(2x + 1) - 3(x - 3)$. 1 mark for the correct answer.

7 (4 marks)

Simplify fully $\frac{2x^2 + 5x - 3}{x^2 - 16} \div \frac{4x - 2}{x^2 + 4x}$

To divide, flip the second fraction and multiply:

$$\frac{2x^2 + 5x - 3}{x^2 - 16} \times \frac{x^2 + 4x}{4x - 2}$$

Factorise everything:

$$2x^2 + 5x - 3 = (2x - 1)(x + 3) \quad x^2 - 16 = (x + 4)(x - 4)$$

$$x^2 + 4x = x(x + 4) \quad 4x - 2 = 2(2x - 1)$$

$$\frac{(2x - 1)(x + 3)}{(x + 4)(x - 4)} \times \frac{x(x + 4)}{2(2x - 1)}$$

Cancel $(2x - 1)$ and $(x + 4)$.

Answer: $\frac{x(x + 3)}{2(x - 4)}$ (or $\frac{x^2 + 3x}{2x - 8}$)

Tip: Factorise first, then flip and cancel. Multiplying everything out gives a quartic over a cubic, which is very hard to simplify.

Marks: 1 mark for flipping the second fraction and multiplying. 1 mark for factorising the two quadratics. 1 mark for factorising $x^2 + 4x$ and $4x - 2$. 1 mark for the correct answer.

8 (4 marks)

Show that

$$\frac{x}{x-3} - \frac{5x+6}{x^2+x-12} = \frac{x+2}{x+4}$$

Factorise the quadratic denominator first: $x^2 + x - 12 = (x - 3)(x + 4)$

So the common denominator is $(x - 3)(x + 4)$.

$$\frac{x}{x-3} = \frac{x(x+4)}{(x-3)(x+4)}$$

$$\text{Left side} = \frac{x(x+4) - (5x+6)}{(x-3)(x+4)}$$

$$= \frac{x^2 + 4x - 5x - 6}{(x-3)(x+4)}$$

$$= \frac{x^2 - x - 6}{(x-3)(x+4)}$$

Factorise the numerator: $x^2 - x - 6 = (x - 3)(x + 2)$

$$= \frac{(x-3)(x+2)}{(x-3)(x+4)}$$

Cancel $(x - 3)$: $= \frac{x+2}{x+4}$, as required.

Answer: Left side $= \frac{(x-3)(x+2)}{(x-3)(x+4)} = \frac{x+2}{x+4}$

Tip: It's a "show that", so write every step. Bracket $(5x + 6)$ when you subtract it, or you get $+6$ instead of -6 .

Marks: 1 mark for factorising $x^2 + x - 12$. 1 mark for a correct single fraction. 1 mark for the numerator $x^2 - x - 6$. 1 mark for factorising it and cancelling to reach the result.

9 (5 marks)

A cyclist rides 12 km at a steady speed of x km/h.

She then rides a further 15 km at a steady speed of $(x + 3)$ km/h.

The total time for the whole journey is 2 hours.

(a) Show that $2x^2 - 21x - 36 = 0$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

Time for the first part = $\frac{12}{x}$ hours. Time for the second part = $\frac{15}{x+3}$ hours.

The total time is 2 hours, so $\frac{12}{x} + \frac{15}{x+3} = 2$

Multiply every term by $x(x+3)$:

$$12(x+3) + 15x = 2x(x+3)$$

$$12x + 36 + 15x = 2x^2 + 6x$$

$$27x + 36 = 2x^2 + 6x$$

$$0 = 2x^2 + 6x - 27x - 36$$

$$2x^2 - 21x - 36 = 0, \text{ as required.}$$

Answer: $\frac{12}{x} + \frac{15}{x+3} = 2$ leads to $2x^2 - 21x - 36 = 0$

Tip: Multiply **every** term by $x(x+3)$, including the 2 on the right-hand side. Forgetting to do this is the most common slip.

Marks: 1 mark for $\frac{12}{x}$ and $\frac{15}{x+3}$. 1 mark for clearing the fractions correctly. 1 mark for reaching the given equation with no errors.

- (b) Work out the cyclist's speed for the 15 km part of the journey.

Factorise: $2 \times (-36) = -72$, and -24 and 3 multiply to -72 and add to -21 .

$$2x^2 - 24x + 3x - 36 = 2x(x-12) + 3(x-12) = (2x+3)(x-12)$$

$$\text{So } x = 12 \text{ or } x = -\frac{3}{2}$$

A speed cannot be negative, so $x = 12$.

The speed for the 15 km part is $x+3 = 15$ km/h.

Answer: 15 km/h

Tip: Read the question: it asks for the speed on the **second** part, which is $x+3$, not x .

Marks: 1 mark for solving to get $x = 12$ (you may ignore $-\frac{3}{2}$). 1 mark for 15.