

Volume and Surface Area of Prisms and Cylinders

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

A cylinder has radius 4 cm and height 9 cm.

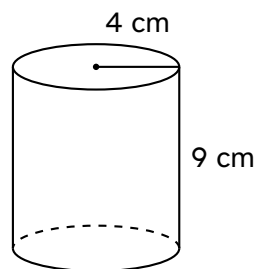


Diagram **NOT** accurately drawn

Work out the volume of the cylinder.

Give your answer correct to 3 significant figures.

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$= \pi \times 4^2 \times 9$$

$$= 144\pi = 452.3 \dots$$

$$= 452 \text{ (3 s.f.)}$$

Answer: 452 cm³

Marks: 1 mark for $\pi \times 4^2 \times 9$. 1 mark for 452.

2 (3 marks)

A water trough is a prism.

Its cross section is a trapezium with parallel sides of 40 cm and 30 cm.

The trough is 120 cm long.

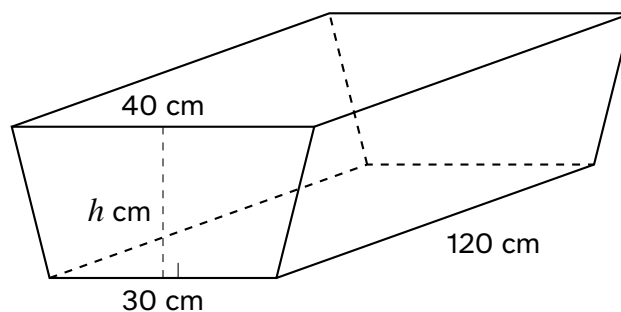


Diagram **NOT** accurately drawn

The trough holds 84 litres of water when it is full.

1 litre = 1000 cm³

Work out the value of h .

Volume in cm³: $84 \times 1000 = 84\,000 \text{ cm}^3$

Volume of a prism = area of cross section $\times$ length

Area of cross section = $84\,000 \div 120 = 700 \text{ cm}^2$

The cross section is a trapezium: $\frac{1}{2} \times (40 + 30) \times h = 700$

$$35h = 700$$

$$h = 20$$

Answer: 20

Tip: Change litres to cm³ first ($\times 1000$), so that all the units match. The area of a trapezium is $\frac{1}{2}(a + b)h$.

Marks: 1 mark for $84\,000 \div 120$ (or 700). 1 mark for $\frac{1}{2} \times (40 + 30) \times h = 700$. 1 mark for 20.

3 (2 marks)

A tin of soup is a cylinder with diameter 7.5 cm and height 11 cm.

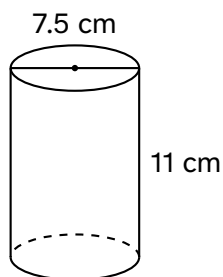


Diagram **NOT** accurately drawn

The label covers the curved surface of the tin.

Work out the area of the label.

Give your answer correct to 3 significant figures.

The label unrolls into a rectangle.

Its length is the circumference, πd , and its width is the height.

$$\text{Area} = \pi \times 7.5 \times 11 = 259.1\dots$$

$$= 259 \text{ (3 s.f.)}$$

Answer: 259 cm²

Marks: 1 mark for $\pi \times 7.5 \times 11$. 1 mark for 259.

4 (4 marks)

A solid cylinder has radius r cm and height $2r$ cm.

The total surface area is 150π cm².

Find the volume of the cylinder. Give your answer in terms of π .

Total surface area = two ends + curved surface

$$\text{Two ends: } 2\pi r^2$$

$$\text{Curved surface: } 2\pi r \times 2r = 4\pi r^2$$

$$\text{Total: } 2\pi r^2 + 4\pi r^2 = 6\pi r^2$$

$$6\pi r^2 = 150\pi$$

$$r^2 = 25, \text{ so } r = 5$$

$$\text{Height} = 2r = 10$$

$$\text{Volume} = \pi \times 5^2 \times 10 = 250\pi$$

Answer: $250\pi \text{ cm}^3$

Tip: Replace the height with $2r$ straight away, so every term is in r . The question asks for the volume, so don't stop at $r = 5$.

Marks: 1 mark for $2\pi r^2 + 4\pi r^2$. 1 mark for $6\pi r^2 = 150\pi$. 1 mark for $r = 5$. 1 mark for 250π .

5 (4 marks)

A swimming pool is a prism. Its cross section is a trapezium.

The pool is 25 m long and 10 m wide. The depth is 1 m at the shallow end and 3 m at the deep end.

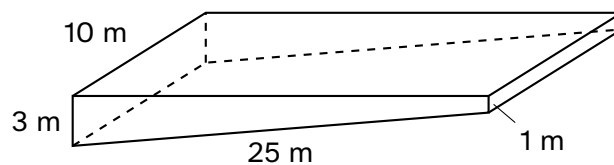


Diagram **NOT** accurately drawn

Water flows into the empty pool at a rate of 0.5 m^3 per minute.

How long will it take to fill the pool? Give your answer in hours, correct to 3 significant figures.

The cross section is a trapezium.

$$\text{Area} = \frac{1}{2}(a + b)h = \frac{1}{2} \times (1 + 3) \times 25 = 50 \text{ m}^2$$

$$\text{Volume} = \text{area} \times \text{width} = 50 \times 10 = 500 \text{ m}^3$$

$$\text{Time} = 500 \div 0.5 = 1000 \text{ minutes}$$

$$\text{Change to hours: } 1000 \div 60 = 16.66 \dots$$

= 16.7 hours (3 s.f.)

Answer: 16.7 hours

Tip: The answer must be in hours. Divide the minutes by 60 (not 100).

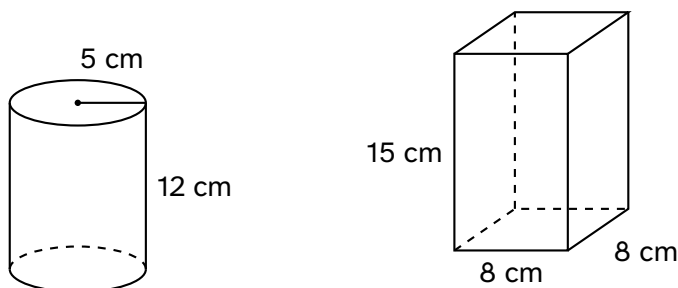
Marks: 1 mark for $\frac{1}{2}(1 + 3) \times 25$. 1 mark for $\times 10$. 1 mark for $\div 0.5$ and $\div 60$. 1 mark for 16.7.

6 (5 marks)

A company is choosing between two designs for a closed tin.

Design A is a cylinder with radius 5 cm and height 12 cm.

Design B is a cuboid with a square base of side 8 cm and a height of 15 cm.



Design A

Design B

*Diagram **NOT** accurately drawn*

The tin must hold at least 900 cm^3 .

The company will choose the design that holds at least 900 cm^3 and uses the smaller area of metal.

Which design should the company choose?

You must show how you get your answer.

Volumes

Design A: $\pi \times 5^2 \times 12 = 300\pi = 942.4 \dots \text{ cm}^3$

Design B: $8 \times 8 \times 15 = 960 \text{ cm}^3$

Both designs hold at least 900 cm^3 .

Areas of metal (total surface areas)

Design A: two ends $2 \times \pi \times 5^2 = 50\pi$, curved surface $2 \times \pi \times 5 \times 12 = 120\pi$

Total for A = $170\pi = 534.0 \dots \text{ cm}^2$

Design B: two square ends $2 \times 8 \times 8 = 128$, four sides $4 \times 8 \times 15 = 480$

$$\text{Total for B} = 128 + 480 = 608 \text{ cm}^2$$

534.0 ... < 608, so Design A uses less metal.

Answer: Design A (both hold at least 900 cm^3 ; A uses 534 cm^2 of metal, B uses 608 cm^2)

Tip: Check both conditions: the volume **and** the area of metal. A closed tin has a top and a bottom, so include both ends.

Marks: 1 mark for both volumes. 1 mark for A's ends or curved surface. 1 mark for A's total (534). 1 mark for B's total (608). 1 mark for Design A with correct figures.

7 (4 marks)

The diagram shows a solid cylinder with radius 3 cm and height h cm.

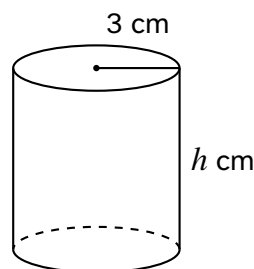


Diagram **NOT** accurately drawn

The cylinder is made from aluminium.

The density of aluminium is 2.7 g/cm^3

The mass of the cylinder is 1 kg.

Work out the height, h cm, of the cylinder.

Give your answer correct to 3 significant figures.

Change the mass to grams: $1 \text{ kg} = 1000 \text{ g}$

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{1000}{2.7} = 370.37 \dots \text{ cm}^3$$

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$\pi \times 3^2 \times h = 370.37 \dots$$

$$h = 370.37 \dots \div (9\pi) = 13.09 \dots$$

$$h = 13.1 \text{ (3 s.f.)}$$

Answer: 13.1 cm

Tip: Change kilograms to grams first, because the density is in g/cm^3 . Then $\text{volume} = \text{mass} \div \text{density}$.

Marks: 1 mark for $1000 \div 2.7$. 1 mark for $\pi \times 3^2 \times h$. 1 mark for dividing to find h . 1 mark for 13.1.
