

Vector Proof

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

$$\overrightarrow{PQ} = 3\mathbf{a} - 6\mathbf{b} \text{ and } \overrightarrow{RS} = \mathbf{a} - 2\mathbf{b}$$

Explain why PQ is parallel to RS .

Take out a factor of 3: $3\mathbf{a} - 6\mathbf{b} = 3(\mathbf{a} - 2\mathbf{b})$

$$\text{So } \overrightarrow{PQ} = 3\overrightarrow{RS}$$

One vector is a multiple of the other, so PQ is parallel to RS .

Answer: $\overrightarrow{PQ} = 3\overrightarrow{RS}$, so PQ is parallel to RS .

Tip: Say “ $\overrightarrow{PQ}$ is a multiple of $\overrightarrow{RS}$ ”. Words like “same gradient” or “both have $\mathbf{a}$ and $\mathbf{b}$ ” do not get the mark.

Marks: 1 mark for $3(\mathbf{a} - 2\mathbf{b})$. 1 mark for saying it is a multiple, so parallel.

2 (2 marks)

$$\overrightarrow{AB} = 4\mathbf{p} + 2\mathbf{q} \text{ and } \overrightarrow{BC} = \mathbf{p} + k\mathbf{q}$$

A , B and C lie on a straight line.

Find the value of k .

On a straight line, $\overrightarrow{BC}$ must be a multiple of $\overrightarrow{AB}$.

The $\mathbf{p}$ part of $\overrightarrow{BC}$ is $\frac{1}{4}$ of the $\mathbf{p}$ part of $\overrightarrow{AB}$.

$$\text{So } \overrightarrow{BC} = \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}(4\mathbf{p} + 2\mathbf{q}) = \mathbf{p} + \frac{1}{2}\mathbf{q}$$

Compare with $\mathbf{p} + k\mathbf{q}$: $k = \frac{1}{2}$

Answer: $k = \frac{1}{2}$

Marks: 1 mark for $\overrightarrow{BC} = \frac{1}{4}\overrightarrow{AB}$. 1 mark for $\frac{1}{2}$.

3 (3 marks)

OPQ is a triangle. $\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$

R is the point on PQ such that $PR:RQ = 1:2$

Find $\overrightarrow{OR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Give your answer in its simplest form.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$$

$$PR:RQ = 1:2 \text{ is 3 parts, so } \overrightarrow{PR} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$$

$$= \mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$$

$$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$$

Answer: $\overrightarrow{OR} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Tip: Watch the direction: $\overrightarrow{PQ} = \mathbf{b} - \mathbf{a}$, not $\mathbf{a} - \mathbf{b}$. Put it in brackets when you multiply by $\frac{1}{3}$.

Marks: 1 mark for $\overrightarrow{PQ} = \mathbf{b} - \mathbf{a}$. 1 mark for $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$. 1 mark for $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

4 (5 marks)

$OACB$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

M is the point on AC such that $AM:MC = 3:2$

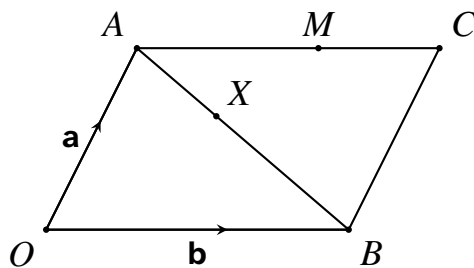


Diagram **NOT** accurately drawn

- (a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, the vector $\overrightarrow{OM}$

$OACB$ is a parallelogram, so $\overrightarrow{AC} = \overrightarrow{OB} = \mathbf{b}$

$AM:MC = 3:2$ is 5 parts, so $\overrightarrow{AM} = \frac{3}{5}\mathbf{b}$

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{3}{5}\mathbf{b}$$

Answer: $\overrightarrow{OM} = \mathbf{a} + \frac{3}{5}\mathbf{b}$

- (b) X is the point on AB such that $AX:XB = 3:5$
Show that O , X and M lie on a straight line.

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

$AX:XB = 3:5$ is 8 parts, so $\overrightarrow{AX} = \frac{3}{8}(\mathbf{b} - \mathbf{a})$

$$\overrightarrow{OX} = \mathbf{a} + \frac{3}{8}(\mathbf{b} - \mathbf{a}) = \frac{5}{8}\mathbf{a} + \frac{3}{8}\mathbf{b}$$

$$\text{Take out } \frac{5}{8}: \overrightarrow{OX} = \frac{5}{8}\left(\mathbf{a} + \frac{3}{5}\mathbf{b}\right)$$

$$\text{So } \overrightarrow{OX} = \frac{5}{8}\overrightarrow{OM}$$

OX is parallel to OM , and they share the point O .

So O , X and M lie on a straight line.

Tip: $AX:XB = 3:5$ means AX is $\frac{3}{8}$ of AB (3 out of 8 parts), not $\frac{3}{5}$.

Marks: 1 mark for $\overrightarrow{AB}$. 1 mark for $\overrightarrow{OX}$. 1 mark for showing $\overrightarrow{OX}$ is a multiple of $\overrightarrow{OM}$. 1 mark for the conclusion (parallel and a common point).

5 (4 marks)

ABC is a triangle.

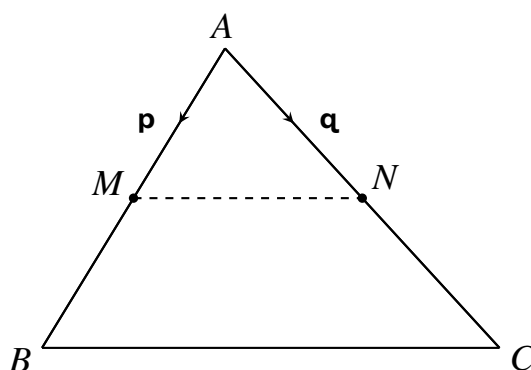


Diagram **NOT** accurately drawn

M is the midpoint of AB and N is the midpoint of AC .

$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$

Use a vector method to prove that MN is parallel to BC .

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\mathbf{p} + \mathbf{q} = \mathbf{q} - \mathbf{p}$$

$$\overrightarrow{MA} = -\frac{1}{2}\mathbf{p} \text{ and } \overrightarrow{AN} = \frac{1}{2}\mathbf{q}$$

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN} = -\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$$

$$= \frac{1}{2}(\mathbf{q} - \mathbf{p})$$

$$\text{So } \overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}$$

$\overrightarrow{MN}$ is a multiple of $\overrightarrow{BC}$, so MN is parallel to BC .

Marks: 1 mark for $\overrightarrow{BC}$. 1 mark for a path for $\overrightarrow{MN}$. 1 mark for $\frac{1}{2}(\mathbf{q} - \mathbf{p})$. 1 mark for $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}$, so parallel.

6 (4 marks)

OAB is a triangle.

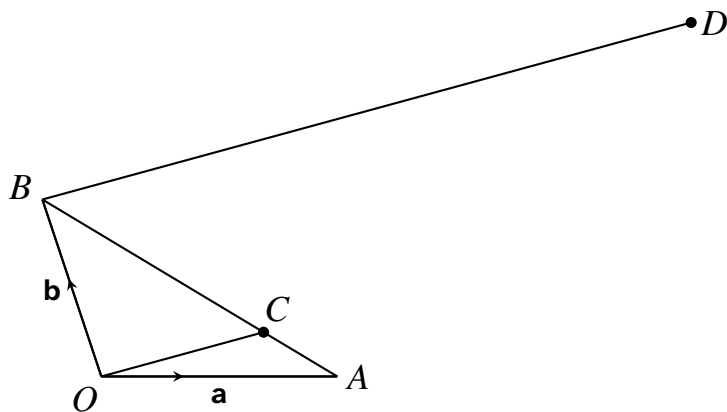


Diagram **NOT** accurately drawn

$$\overrightarrow{OA} = \mathbf{a} \quad \overrightarrow{OB} = \mathbf{b}$$

C is the point on AB such that $AC:CB = 1:3$

D is the point such that $\overrightarrow{OD} = 3\mathbf{a} + 2\mathbf{b}$

Prove, using vectors, that OC is parallel to BD .

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$$

$$AC:CB = 1:3, \text{ so } \overrightarrow{AC} = \frac{1}{4}\overrightarrow{AB} = -\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$$

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{a} - \frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} = \frac{1}{4}(3\mathbf{a} + \mathbf{b})$$

$$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD} = -\mathbf{b} + 3\mathbf{a} + 2\mathbf{b} = 3\mathbf{a} + \mathbf{b}$$

$$\text{So } \overrightarrow{BD} = 4\overrightarrow{OC}$$

$\overrightarrow{BD}$ is a multiple of $\overrightarrow{OC}$, so OC is parallel to BD .

Tip: $AC:CB = 1:3$ means AC is $\frac{1}{4}$ of AB (4 parts in total), not $\frac{1}{3}$.

Marks: 1 mark for $\overrightarrow{AC}$ (or $\overrightarrow{AB}$ used correctly). 1 mark for $\overrightarrow{OC} = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$. 1 mark for $\overrightarrow{BD} = 3\mathbf{a} + \mathbf{b}$.
1 mark for $\overrightarrow{BD} = 4\overrightarrow{OC}$, so parallel.

7 (5 marks)

OAB is a triangle.

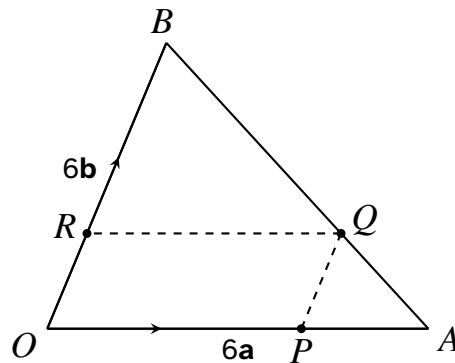


Diagram **NOT** accurately drawn

$$\overrightarrow{OA} = 6\mathbf{a} \quad \overrightarrow{OB} = 6\mathbf{b}$$

P is the point on OA such that $OP:PA = 2:1$

Q is the point on AB such that $AQ:QB = 1:2$

R is the point on OB such that $OR:RB = 1:2$

Use a vector method to prove that $OPQR$ is a parallelogram.

$$\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA} = 4\mathbf{a} \quad \text{and} \quad \overrightarrow{OR} = \frac{1}{3}\overrightarrow{OB} = 2\mathbf{b}$$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -6\mathbf{a} + 6\mathbf{b}$$

$$\overrightarrow{AQ} = \frac{1}{3}\overrightarrow{AB} = -2\mathbf{a} + 2\mathbf{b}$$

$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ} = 2\mathbf{a} - 2\mathbf{a} + 2\mathbf{b} = 2\mathbf{b}, \text{ so } \overrightarrow{PQ} = \overrightarrow{OR}$$

$$\overrightarrow{RQ} = \overrightarrow{RO} + \overrightarrow{OP} + \overrightarrow{PQ} = -2\mathbf{b} + 4\mathbf{a} + 2\mathbf{b} = 4\mathbf{a}, \text{ so } \overrightarrow{RQ} = \overrightarrow{OP}$$

Both pairs of opposite sides are parallel (and equal in length).

So $OPQR$ is a parallelogram.

Tip: Equal vectors are parallel **and** the same length. Show this for a pair of opposite sides (or for both pairs), then name the shape.

Marks: 1 mark for $\overrightarrow{OP} = 4\mathbf{a}$ and $\overrightarrow{OR} = 2\mathbf{b}$. 1 mark for $\overrightarrow{AQ}$ (or $\overrightarrow{OQ} = 4\mathbf{a} + 2\mathbf{b}$). 1 mark for $\overrightarrow{PQ} = 2\mathbf{b}$. 1 mark for $\overrightarrow{RQ} = 4\mathbf{a}$. 1 mark for a complete conclusion.