

Upper and Lower Bounds

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The mass of a bag is 60 kg, correct to the nearest 10 kg.
Write down the error interval for the mass, m kg.

To the nearest 10 kg, so the bounds are half of 10 (5 kg) either side.

Lower bound: $60 - 5 = 55$

Upper bound: $60 + 5 = 65$

65 itself would round up to 70, so use $<$ at the top.

Answer: $55 \leq m < 65$

Marks: 1 mark for 55. 1 mark for 65 (in a correct interval).

2 (2 marks)

The thickness of a sheet of card is 0.26 mm, correct to 2 significant figures.
Complete the error interval for the thickness, t mm.

The second significant figure (6) is in the hundredths column.

So t has been rounded to the nearest 0.01

Half of 0.01 is 0.005

Lower bound: $0.26 - 0.005 = 0.255$

Upper bound: $0.26 + 0.005 = 0.265$

Answer: $0.255 \leq t < 0.265$

Tip: The zero before the decimal point is not significant. 0.26 to 2 significant figures means to the nearest 0.01, not to the nearest 0.1

Marks: 1 mark for 0.255. 1 mark for 0.265.

3 (2 marks)

A stopwatch shows a time in seconds, to 2 decimal places.

Leon writes down the time by truncating it to a whole number of seconds.

He writes down 58 seconds.

Write down the error interval for the time shown on the stopwatch, t seconds.

Truncating means chopping off the digits after the decimal point (no rounding).

Any time from 58.00 up to 58.99 seconds truncates to 58

So t is at least 58 and less than 59.

Answer: $58 \leq t < 59$

Tip: Truncating is not rounding. A time of 57.8 seconds truncates to 57, so the interval does not start at 57.5

Marks: 1 mark for 58. 1 mark for 59 with the correct inequality signs. ($58 \leq t \leq 58.99$ is also correct, because the stopwatch shows 2 decimal places.)

4 (2 marks)

The length of a side of a square is 9 cm, correct to the nearest cm.

Work out the upper bound for the perimeter of the square.

Upper bound of a side: $9 + 0.5 = 9.5$ cm

A square has 4 sides.

Upper bound of the perimeter: $4 \times 9.5 = 38$ cm

Answer: 38 cm

Marks: 1 mark for 4×9.5 . 1 mark for 38.

5 (2 marks)

The circumference of a bicycle wheel is 185 cm, correct to the nearest 5 cm.

Work out the lower bound for the diameter of the wheel.

Give your answer correct to 1 decimal place.

To the nearest 5 cm, so the bounds are half of 5 (2.5 cm) either side.

Lower bound of the circumference: $185 - 2.5 = 182.5$ cm

Circumference = πd , so $d = \frac{\text{circumference}}{\pi}$

Lower bound of the diameter: $\frac{182.5}{\pi} = 58.091 \dots$

To 1 decimal place: 58.1 cm

Answer: 58.1 cm

Tip: “To the nearest 5 cm” means 2.5 cm either side, not 5 cm either side.

Marks: 1 mark for 182.5. 1 mark for 58.1.

6 (3 marks)

A shelf in a shop can safely hold a load of 180 kg, correct to the nearest 10 kg.

Tins of paint are put on the shelf.

Each tin weighs 4.6 kg, correct to 1 decimal place.

Work out the greatest number of tins that can **always** be put on the shelf safely.

You must show your working.

“Always safely” means plan for the worst case:

the safe load is as small as it could be,

and each tin is as heavy as it could be.

Lower bound of the safe load: $180 - 5 = 175$ kg

Upper bound of one tin: $4.6 + 0.05 = 4.65$ kg

$175 \div 4.65 = 37.6 \dots$

38 tins might be too heavy, so round down to 37.

Answer: 37

Tip: Choose each bound by asking which value makes things worst. Here, use the **lower** bound of the safe load and the **upper** bound of a tin. Then round **down**.

Marks: 1 mark for 175 and 4.65. 1 mark for $175 \div 4.65$. 1 mark for 37.

7 (3 marks)

ABC is a right-angled triangle.

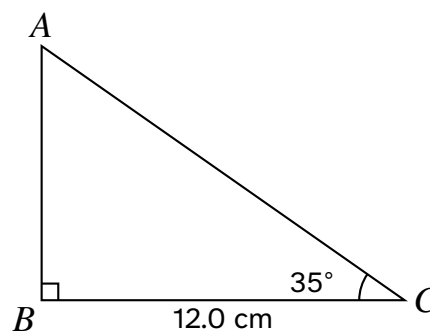


Diagram **NOT** accurately drawn

$BC = 12.0$ cm correct to the nearest mm.

Angle $ACB = 35^\circ$ correct to the nearest degree.

Calculate the upper bound for the length of AB .

Give your answer correct to 3 significant figures.

You must show all your working.

AB is opposite the 35° angle and BC is adjacent to it, so use $\tan$.

$$AB = BC \times \tan C$$

Nearest mm = nearest 0.1 cm, so the upper bound of BC is 12.05 cm.

Nearest degree, so the upper bound of angle ACB is 35.5° .

$\tan$ gets bigger as the angle gets bigger, so use both upper bounds.

$$AB = 12.05 \times \tan 35.5^\circ = 8.595 \dots$$

To 3 significant figures: 8.60 cm

Answer: 8.60 cm

Tip: The angle has bounds too: 35° to the nearest degree means 34.5° to 35.5° . Make sure your calculator is in degrees.

Marks: 1 mark for 12.05 or 35.5° . 1 mark for $12.05 \times \tan 35.5^\circ$. 1 mark for 8.60 (or an answer from 8.59 to 8.60).

8 (4 marks)

The kinetic energy, E joules, of an object of mass m kg moving at a speed of v m/s is given by the formula

$$E = \frac{1}{2}mv^2$$

A ball has mass 0.45 kg, correct to 2 significant figures.

The kinetic energy of the ball is 32 joules, correct to 2 significant figures.

Work out the upper bound and the lower bound for the speed of the ball.

Give your answers correct to 1 decimal place.

You must show all your working.

Make v the subject.

Multiply by 2: $2E = mv^2$

Divide by m : $v^2 = \frac{2E}{m}$

Square root: $v = \sqrt{\frac{2E}{m}}$

Bounds of E : $31.5 \leq E < 32.5$

Bounds of m : $0.445 \leq m < 0.455$

Upper bound of v : largest E and smallest m

$$\sqrt{\frac{2 \times 32.5}{0.445}} = 12.085 \dots = 12.1$$

Lower bound of v : smallest E and largest m

$$\sqrt{\frac{2 \times 31.5}{0.455}} = 11.766 \dots = 11.8$$

Answer: UB = 12.1, LB = 11.8

Tip: m is on the bottom of the fraction, so the upper bound of v uses the **lower** bound of m . Using both upper bounds is a common mistake.

Marks: 1 mark for the bounds of E or m . 1 mark for rearranging to $v = \sqrt{\frac{2E}{m}}$ (or equivalent). 1 mark for using the upper bound of E with the lower bound of m (or the other way round). 1 mark for both answers.

9 (5 marks)

A tin is in the shape of a cylinder.

The volume of the tin is 850 cm^3 , correct to 3 significant figures.

The height of the tin is 12.4 cm, correct to 1 decimal place.

By considering bounds, work out the radius of the tin to a suitable degree of accuracy.

Give a reason for your answer.

Volume of a cylinder: $V = \pi r^2 h$, so $r = \sqrt{\frac{V}{\pi h}}$

Bounds of V : 849.5 and 850.5

Bounds of h : 12.35 and 12.45

Upper bound of r : largest V and smallest h

$$\sqrt{\frac{850.5}{\pi \times 12.35}} = 4.681 \dots$$

Lower bound of r : smallest V and largest h

$$\sqrt{\frac{849.5}{\pi \times 12.45}} = 4.660 \dots$$

To 3 significant figures these are 4.68 and 4.66, which are different.

To 2 significant figures both are 4.7, which agree.

So the radius is 4.7 cm, because the upper and lower bounds both round to 4.7 to 2 significant figures.

Answer: 4.7 cm (both bounds round to 4.7 to 2 s.f.)

Tip: 850 correct to 3 significant figures means to the nearest 1, so the bounds are 849.5 and 850.5, not 845 and 855.

Marks: 1 mark for the bounds of V and h . 1 mark for the upper bound of r . 1 mark for the lower bound of r . 1 mark for 4.7. 1 mark for the reason.
