

Trigonometric Graphs

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

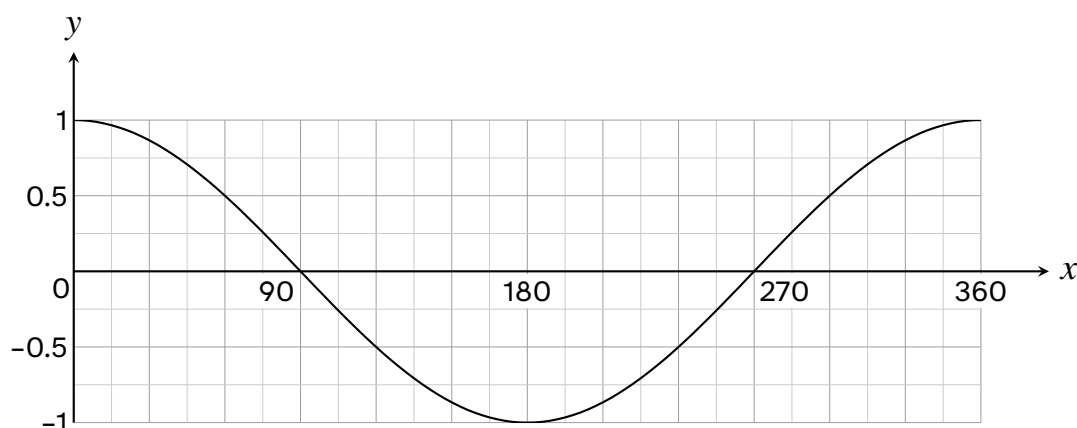
1 (4 marks)

- (a) On the grid, sketch the graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$

Plot the key points of $y = \cos x^\circ$:

$(0, 1)$, $(90, 0)$, $(180, -1)$, $(270, 0)$, $(360, 1)$

Join them with a smooth wave-shaped curve.



Answer: Cosine curve through $(0, 1)$, $(90, 0)$, $(180, -1)$, $(270, 0)$, $(360, 1)$

Tip: The curve should be smooth and rounded at the top and bottom, not pointed. It must reach exactly 1 and -1 .

Marks: 2 marks for a fully correct curve. 1 mark for the correct shape or 3 correct key points.

- (b) Use your graph to find estimates of the solutions of $\cos x^\circ = 0.3$ for $0 \leq x \leq 360$

Draw the line $y = 0.3$ across your graph.

It crosses the curve twice, so there are two solutions.

Reading from the graph, the first crossing is at about $x = 73$.

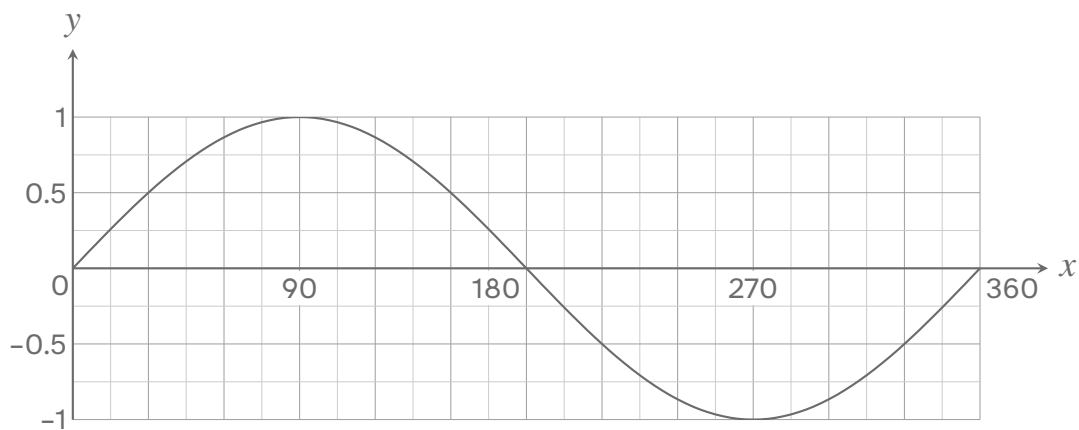
The cosine graph is symmetrical about $x = 180$, so the other is about $360 - 73 = 287$.

Answer: About 73 and 287

Marks: 1 mark for each solution (answers close to these are accepted).

2 (7 marks)

Here is a graph of $y = \sin x^\circ$ for $0 \leq x \leq 360$



- (a) One solution of $\sin x^\circ = -0.4$ is $x = 203.6$ to 1 decimal place.
Find the other solution for $0 \leq x \leq 360$

Between 180 and 360, the sine graph is symmetrical about $x = 270$.

203.6 is $270 - 203.6 = 66.4$ to the left of 270.

So the other solution is 66.4 to the right: $270 + 66.4 = 336.4$

(Or: $203.6 = 180 + 23.6$, so the other solution is $360 - 23.6 = 336.4$.)

Answer: 336.4

Tip: Use a quick sketch to see where the line of symmetry is. Here it is $x = 270$, where $\sin x^\circ$ has its minimum.

Marks: 1 mark for a correct method using symmetry. 1 mark for 336.4.

- (b) Solve the equation $2 \sin x^\circ = -\sqrt{2}$ for $0 \leq x \leq 360$

Divide both sides by 2: $\sin x^\circ = -\frac{\sqrt{2}}{2}$

Calculator: $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ$, which is outside the range.

$\sin 45^\circ = \frac{\sqrt{2}}{2}$, so use the graph to find where $\sin x^\circ = -\frac{\sqrt{2}}{2}$:

$$180 + 45 = 225 \quad \text{and} \quad 360 - 45 = 315$$

Answer: 225 and 315

Tip: The calculator gives -45 , which is not in the range. Use the symmetry of the graph to find the two solutions between 180 and 360, where $\sin x^\circ$ is negative.

Marks: 1 mark for $\sin x^\circ = -\frac{\sqrt{2}}{2}$ (or $x = -45$ seen). 1 mark for both solutions.

- (c) Find all the solutions of $\sin x^\circ = -0.4$ for $360 \leq x \leq 720$

Give your answers correct to 1 decimal place.

From part (a), the solutions for $0 \leq x \leq 360$ are $x = 203.6$ and $x = 336.4$

The sine graph repeats every 360° , so add 360 to each:

$$203.6 + 360 = 563.6 \quad \text{and} \quad 336.4 + 360 = 696.4$$

Answer: 563.6 and 696.4

Tip: The range here starts at $x = 360$, so answers such as 203.6 are not allowed. Add 360 to the solutions you already have.

Marks: 1 mark for one correct solution. 1 mark for both.

- (d) The graph of $y = \sin x^\circ$ is translated by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

Write down the coordinates of the minimum point of the translated graph for $0 \leq x \leq 360$

The minimum point of $y = \sin x^\circ$ is $(270, -1)$.

The vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ moves every point 3 units up and does not move it across.

$$(270, -1 + 3) = (270, 2)$$

Answer: (270, 2)

Tip: The top number of the vector moves the graph across and the bottom number moves it up or down. Only the y-coordinate changes here.

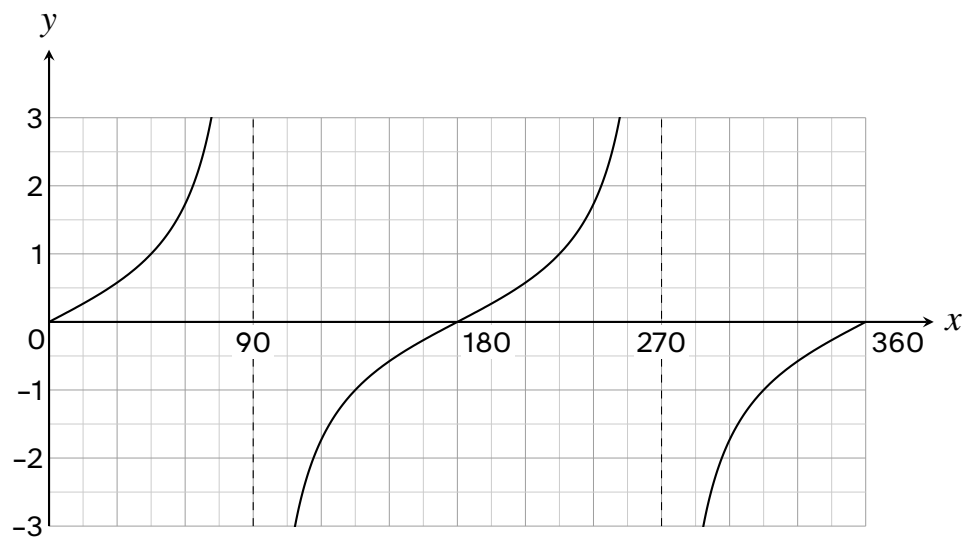
3 (5 marks)

- (a) On the grid, sketch the graph of $y = \tan x^\circ$ for $0 \leq x \leq 360$

$y = \tan x^\circ$ passes through (0, 0), (180, 0) and (360, 0).

$\tan 90^\circ$ and $\tan 270^\circ$ are undefined, so there are asymptotes at $x = 90$ and $x = 270$.

Each branch goes up from left to right and gets closer to the asymptotes without touching them.



Answer: Tan curve through (0, 0), (180, 0), (360, 0) with asymptotes at $x = 90$ and $x = 270$

Marks: 2 marks for a fully correct sketch. 1 mark for one correct branch or the correct asymptotes.

- (b) $\tan 60^\circ = \sqrt{3}$
Find another value of x , for $0 \leq x \leq 360$, for which $\tan x^\circ = \sqrt{3}$

The tan graph repeats every 180° .

$$60 + 180 = 240$$

Answer: 240

- (c) Solve $\tan x^\circ = -1$ for $0 \leq x \leq 360$

Calculator: $\tan^{-1}(-1) = -45^\circ$, which is outside the range.

Tan repeats every 180° , so add 180: $-45 + 180 = 135$

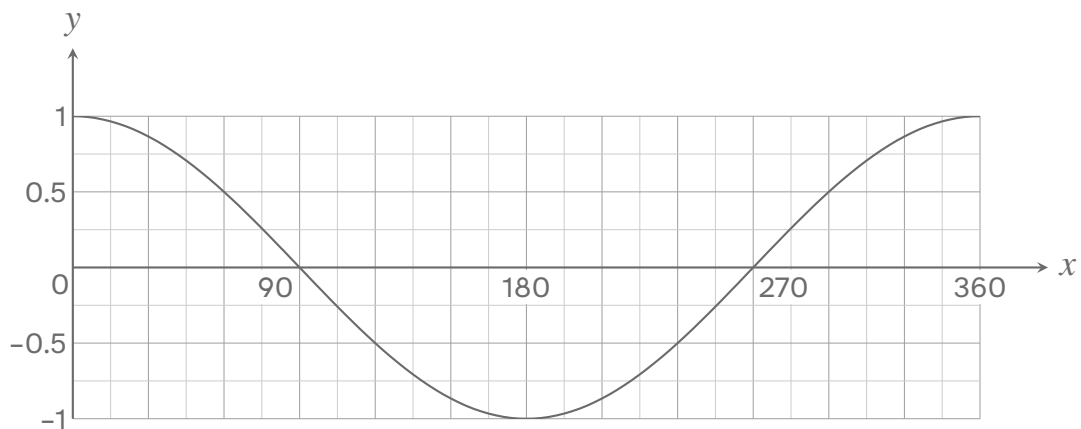
Add 180 again: $135 + 180 = 315$

Answer: 135 and 315

Marks: 1 mark for each correct solution.

4 (2 marks)

Here is a graph of $y = \cos x^\circ$ for $0 \leq x \leq 360$



- (a) Write down the number of solutions of $\cos x^\circ = -0.9$ for $0 \leq x \leq 360$

Imagine the horizontal line $y = -0.9$ across the graph.

It crosses the curve twice, once either side of the minimum point $(180, -1)$.

Answer: 2

- (b) The equation $\cos x^\circ = k$ has exactly **one** solution for $0 \leq x \leq 360$
Write down the value of k .

A horizontal line $y = k$ usually crosses the curve twice.

$y = 1$ touches the curve at $x = 0$ and at $x = 360$, which is two solutions.

$y = -1$ touches the curve only at the minimum point, $x = 180$, which is one solution.

Answer: $k = -1$

Tip: Check both ends of the range. $\cos x^\circ = 1$ at $x = 0$ **and** at $x = 360$, so $k = 1$ gives two solutions, not one.

5 (1 mark)

The graph of $y = \sin x^\circ$ has a maximum point at $(90, 1)$

Write down the coordinates of the maximum point of $y = \sin x^\circ$ for $360 \leq x \leq 720$

The sine graph repeats every 360° .

So the next maximum is at $x = 90 + 360 = 450$, still with $y = 1$.

Answer: $(450, 1)$

6 (4 marks)

The diagram shows a sketch of the curve with equation $y = a \sin x^\circ + b$ for $0 \leq x \leq 360$

The curve passes through the point $(0, 3)$ and has a maximum point at $(90, 7)$.

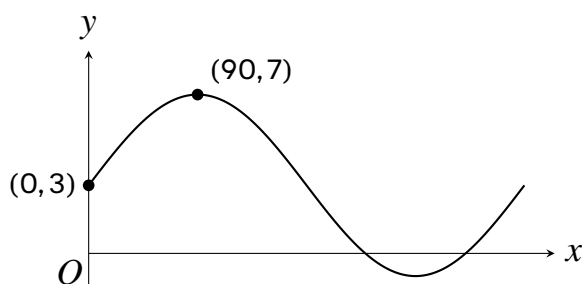


Diagram **NOT** accurately drawn

Find the exact value of y when $x = 240$

Give your answer in the form $p + q\sqrt{3}$ where p and q are integers.

Use $(0, 3)$: $\sin 0^\circ = 0$, so $y = b$.

So $b = 3$.

Use $(90, 7)$: $\sin 90^\circ = 1$, so $y = a + b$.

$a + 3 = 7$, so $a = 4$.

Now $x = 240$: $y = 4 \sin 240^\circ + 3$

$240 = 180 + 60$, so $\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

$$y = 4 \times \left(-\frac{\sqrt{3}}{2}\right) + 3 = 3 - 2\sqrt{3}$$

Answer: $3 - 2\sqrt{3}$

Tip: $\sin 240^\circ$ is negative (the sine graph is below the x -axis between 180° and 360°). Learn $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and use the symmetry of the graph for the sign.

Marks: 1 mark for $b = 3$. 1 mark for $a = 4$. 1 mark for $\sin 240^\circ = -\frac{\sqrt{3}}{2}$. 1 mark for $3 - 2\sqrt{3}$.