

Transformations and Invariant Points

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

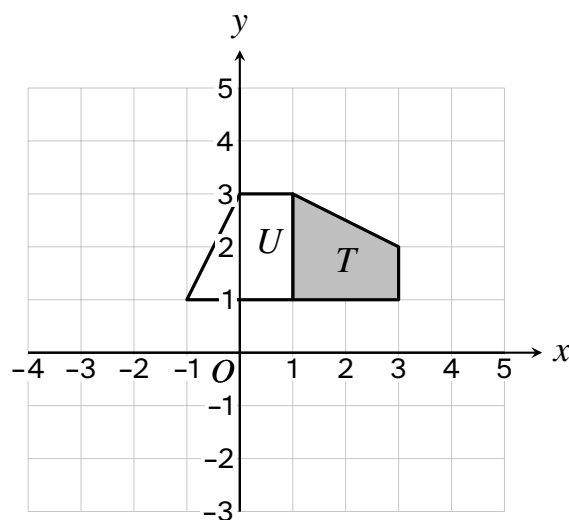
1 (3 marks)

- (a) Rotate shape T 90° anticlockwise about the point $(1, 1)$
Label the image U .

Use tracing paper: trace T , put your pencil on $(1, 1)$ and turn the paper 90° anticlockwise.

Or turn each vertex: from $(1, 1)$, a point 2 right moves to 2 up.

U : $(1, 1)$, $(1, 3)$, $(0, 3)$, $(-1, 1)$



Answer: $(1, 1)$, $(1, 3)$, $(0, 3)$, $(-1, 1)$

Marks: 2 marks for the correct image. 1 mark for a 90° clockwise rotation, or the right orientation in the wrong place.

(b) Write down the coordinates of the invariant point.

An invariant point does not move.

In a rotation, only the centre of rotation stays in the same place.

Answer: (1, 1)

Tip: The invariant point of a rotation is always its centre.

2 (2 marks)

A shape is rotated 90° clockwise about the origin.

The image is then reflected in the x -axis.

Describe fully the single transformation that is equivalent to these two transformations.

Follow a general point (x, y) .

Rotating 90° clockwise about the origin: $(x, y) \rightarrow (y, -x)$

Reflecting in the x -axis changes the sign of the y -coordinate: $(y, -x) \rightarrow (y, x)$

$(x, y) \rightarrow (y, x)$ swaps the coordinates.

This is a reflection in the line $y = x$.

Check with a point: $(3, 1) \rightarrow (1, -3) \rightarrow (1, 3)$, which is the reflection of $(3, 1)$ in $y = x$ ✓

Answer: Reflection in the line $y = x$

Tip: If you are not sure, draw a small shape on a grid, do both transformations, then compare the first and last shapes.

Marks: 1 mark for “reflection”. 1 mark for the line $y = x$.

3 (3 marks)

Shape A is reflected in the line $x = 1$ to give shape B .

Shape B is then reflected in a line L to give shape C .

Line L is parallel to $x = 1$.

Shape C is the image of shape A under the translation $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$.

- (a) Find an equation of line L .

Reflecting in two parallel lines gives a translation.

The translation moves shapes twice the distance between the two lines.

C is A moved 6 units to the right, so the lines are $6 \div 2 = 3$ units apart.

L is 3 units to the right of $x = 1$, so L is $x = 4$.

Check with the point $(0, 0)$: reflect in $x = 1$ to $(2, 0)$, then in $x = 4$ to $(6, 0)$ ✓

Answer: $x = 4$

Tip: The distance between the mirror lines is half the translation distance, not the whole of it. $x = 7$ is a common wrong answer.

Marks: 1 mark for a correct method, e.g. the lines are 3 apart, or a correct check with a point. 1 mark for $x = 4$.

- (b) Explain why there are no invariant points when shape A is mapped onto shape C .

The single transformation from A to C is a translation.

A translation moves every point 6 units to the right.

So no point stays in the same place.

Answer: It is a translation, which moves every point, so no point stays in the same place.

4 (1 mark)

The line L is reflected in the line $y = x$ and is unchanged.

L is not $y = x$. Write down a possible equation of L .

A line that crosses the mirror line at right angles maps onto itself.

Lines perpendicular to $y = x$ have gradient -1 .

So any line $y = -x + c$ works.

Answer: e.g. $y = -x + 4$ (any line $y = -x + c$)

5 (4 marks)

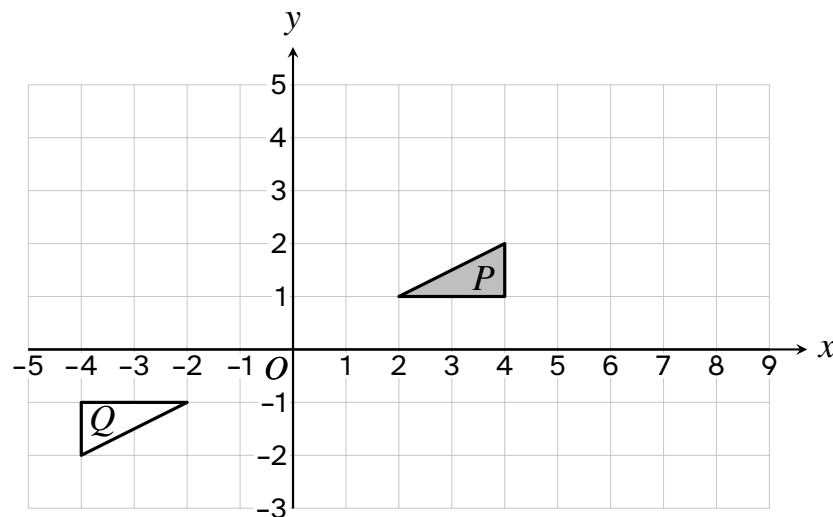
Triangle P is drawn on the grid.

- (a) Enlarge triangle P by scale factor -1 , centre $(0,0)$. Label the image Q .

Scale factor -1 from the origin: each point goes the same distance on the other side of $(0,0)$.

$$(x, y) \rightarrow (-x, -y)$$

$$Q: (-2, -1), (-4, -1), (-4, -2)$$



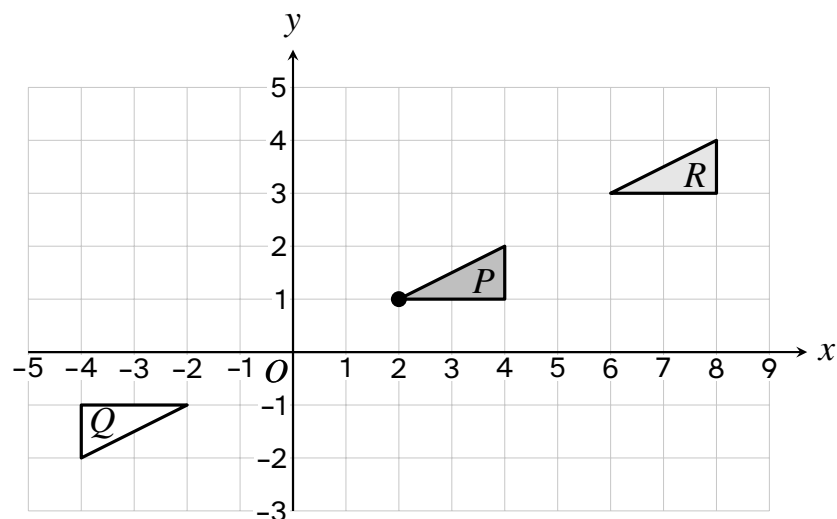
Answer: $Q: (-2, -1), (-4, -1), (-4, -2)$

- (b) Triangle Q is then enlarged by scale factor -1 , centre $(2,1)$, to give triangle R . Describe fully the single transformation that maps triangle P onto triangle R .

Enlarge Q by scale factor -1 from $(2,1)$:

$$(-2, -1) \rightarrow (6, 3), (-4, -1) \rightarrow (8, 3), (-4, -2) \rightarrow (8, 4)$$

$$\text{So } R: (6, 3), (8, 3), (8, 4)$$



R is the same way up as P , and every vertex has moved 4 right and 2 up.

$$(2, 1) \rightarrow (6, 3), (4, 1) \rightarrow (8, 3), (4, 2) \rightarrow (8, 4)$$

Answer: Translation by $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Tip: Two half turns (scale factor -1 twice) put the shape back the right way up, so the result is a translation, not an enlargement.

Marks: 1 mark for “translation”. 1 mark for $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

- (c) Explain why no point is invariant under the transformation that maps triangle P onto triangle R .

The transformation is a translation.

Every point moves 4 right and 2 up, so no point stays where it was.

Answer: It is a translation, so every point moves by the same vector and none stays fixed.

6 (2 marks)

Shape S is transformed by the combined transformation of a reflection in the line $y = 1$ followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

Point $(5, k)$ on shape S is invariant under the combined transformation.
Find the value of k .

Follow a general point (x, y) .

Reflecting in $y = 1$: $(x, y) \rightarrow (x, 2 - y)$

Translating by $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$: $(x, 2 - y) \rightarrow (x, 8 - y)$

An invariant point ends where it started, so $y = 8 - y$

$2y = 8$, so $y = 4$

So $k = 4$. (The combined transformation is a reflection in the line $y = 4$.)

Answer: $k = 4$

Tip: Only the y -coordinate changes here, so any point with $y = 4$ is invariant, whatever its x -coordinate.

Marks: 1 mark for the image of a general point, or for $y = 8 - y$. 1 mark for $k = 4$.

7 (2 marks)

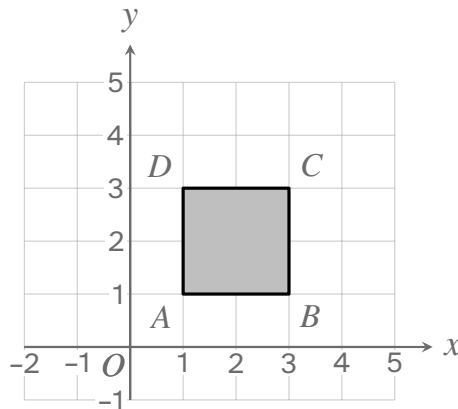
Square $ABCD$ is drawn on the grid.

Square $ABCD$ is rotated 90° anticlockwise about A .

The image is then reflected in a line L .

Under the combined transformation, A and C are both invariant.

Find an equation of line L .



Rotate 90° anticlockwise about $A(1, 1)$.

C is 2 right and 2 up from A . This turns to 2 left and 2 up.

So C moves to $(-1, 3)$. A does not move.

The reflection must send $(-1, 3)$ back to $C(3, 3)$, and keep $A(1, 1)$ where it is.

The mirror line is halfway between $(-1, 3)$ and $(3, 3)$, at right angles to the line joining them: $x = 1$.

$A(1, 1)$ is on $x = 1$, so A stays invariant ✓

Answer: $x = 1$

Tip: Do the transformations in the order given. Draw the rotated square first, then find the line that sends C back to where it started.

Marks: 1 mark for the image of C after the rotation, $(-1, 3)$. 1 mark for $x = 1$.

8 (3 marks)

Triangle P is drawn on the grid.

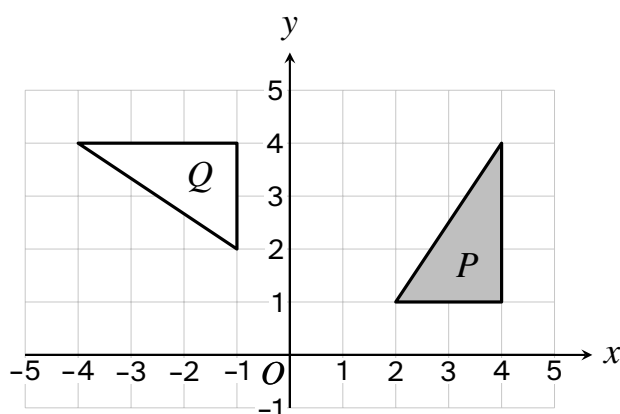
- (a) Rotate triangle P 90° anticlockwise about the origin.
Label the image Q .

Rotating 90° anticlockwise about the origin takes (x, y) to $(-y, x)$.

$$(2, 1) \rightarrow (-1, 2)$$

$$(4, 1) \rightarrow (-1, 4)$$

$$(4, 4) \rightarrow (-4, 4)$$



Answer: $(-1, 2), (-1, 4), (-4, 4)$

Marks: 2 marks for the correct image. 1 mark for a 90° clockwise rotation, or the right orientation in the wrong place.

- (b) Triangle P is rotated 90° anticlockwise about one of its own vertices.
The image of the vertex $(2, 1)$ is $(4, -1)$.
Which vertex of triangle P is invariant?

The invariant vertex is the centre of the rotation.

Try $(4, 1)$: $(2, 1)$ is 2 left of $(4, 1)$.

Turn “2 left” through 90° anticlockwise: it becomes 2 down.

2 down from $(4, 1)$ is $(4, -1)$ ✓

So the centre is $(4, 1)$, and this vertex is invariant.

Answer: $(4, 1)$

Tip: Tracing paper helps: pin each vertex in turn and turn the paper 90° anticlockwise.
