

The Sine Rule

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Here is a triangle.

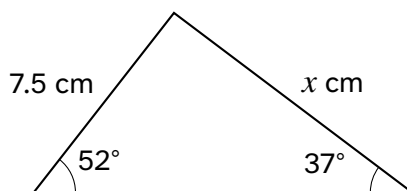


Diagram **NOT** accurately drawn

Work out the value of x .

Give your answer correct to 3 significant figures.

x is opposite the 52° angle.

7.5 is opposite the 37° angle.

Sine rule: $\frac{x}{\sin 52^\circ} = \frac{7.5}{\sin 37^\circ}$

Multiply both sides by $\sin 52^\circ$:

$$x = \frac{7.5 \sin 52^\circ}{\sin 37^\circ} = 9.820 \dots$$

Answer: $x = 9.82$

Marks: 1 mark for a correct sine rule equation. 1 mark for 9.82.

2 (3 marks)

ABC is a triangle.

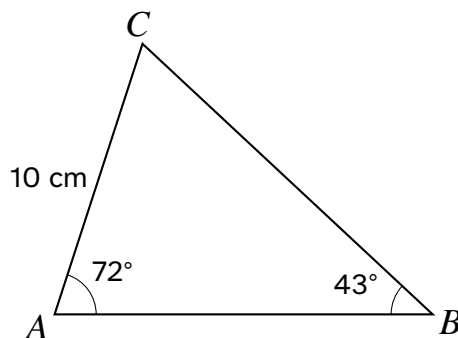


Diagram **NOT** accurately drawn

Work out the length of BC .

Give your answer correct to 3 significant figures.

BC is opposite the 72° angle (at A).

$AC = 10$ cm is opposite the 43° angle (at B).

$$\text{Sine rule: } \frac{BC}{\sin 72^\circ} = \frac{10}{\sin 43^\circ}$$

$$BC = \frac{10 \sin 72^\circ}{\sin 43^\circ} = 13.945 \dots$$

To 3 significant figures: 13.9

Answer: 13.9 cm

Tip: The triangle has no right angle, so do not use SOHCAHTOA or Pythagoras.

Marks: 1 mark for the sine rule. 1 mark for rearranging to $BC = \frac{10 \sin 72^\circ}{\sin 43^\circ}$. 1 mark for 13.9.

3 (3 marks)

Here is a triangle.

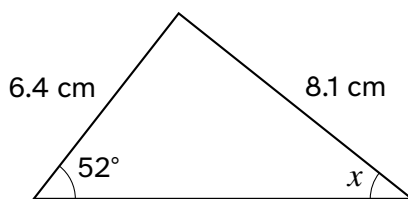


Diagram **NOT** accurately drawn

Work out the size of angle x .

Give your answer correct to 1 decimal place.

x is opposite the 6.4 cm side.

52° is opposite the 8.1 cm side.

To find an angle, put the sines on top: $\frac{\sin x}{6.4} = \frac{\sin 52^\circ}{8.1}$

Multiply both sides by 6.4:

$$\sin x = \frac{6.4 \sin 52^\circ}{8.1} = 0.6226 \dots$$

$$x = \sin^{-1}(0.6226 \dots) = 38.51 \dots$$

Answer: 38.5°

Marks: 1 mark for the sine rule. 1 mark for $\sin x = \frac{6.4 \sin 52^\circ}{8.1}$. 1 mark for 38.5.

4 (3 marks)

In triangle PQR , $PQ = 7.2$ cm, $QR = 9.5$ cm and angle $QPR = 64^\circ$

Find the size of angle PRQ .

Give your answer correct to 1 decimal place.

Sketch the triangle and label it.

$PQ = 7.2$ is opposite angle R (angle PRQ).

$QR = 9.5$ is opposite angle $P = 64^\circ$.

$$\text{Sine rule: } \frac{\sin R}{7.2} = \frac{\sin 64^\circ}{9.5}$$

$$\sin R = \frac{7.2 \sin 64^\circ}{9.5} = 0.6812 \dots$$

$$R = \sin^{-1}(0.6812 \dots) = 42.93 \dots$$

Answer: 42.9°

Tip: Angle PRQ is the angle at R (the middle letter). Pair it with the side opposite it, PQ .

Marks: 1 mark for the sine rule. 1 mark for $\sin R = 0.681 \dots$ 1 mark for 42.9.

5 (5 marks)

The diagram shows a triangular sail PQR .

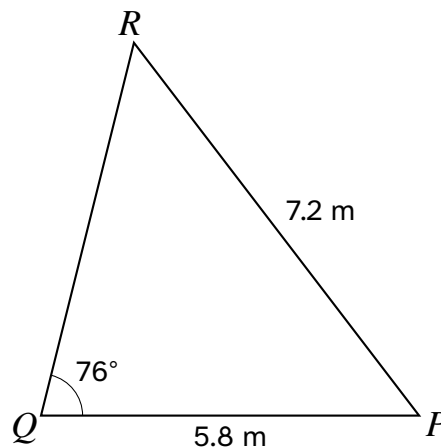


Diagram **NOT** accurately drawn

$PQ = 5.8$ m, $PR = 7.2$ m and angle $PQR = 76^\circ$

Calculate the length of QR .

Give your answer correct to 3 significant figures.

We know PR and the angle opposite it, so start with angle PRQ , which is opposite PQ .

$$\text{Sine rule: } \frac{\sin R}{5.8} = \frac{\sin 76^\circ}{7.2}$$

$$\sin R = \frac{5.8 \sin 76^\circ}{7.2} = 0.7816 \dots, \text{ so angle } PRQ = 51.40 \dots^\circ$$

Angles in a triangle: angle $QPR = 180 - 76 - 51.40 \dots = 52.59 \dots^\circ$

QR is opposite angle QPR .

$$\text{Sine rule: } \frac{QR}{\sin 52.59 \dots^\circ} = \frac{7.2}{\sin 76^\circ}$$

$$QR = \frac{7.2 \sin 52.59 \dots^\circ}{\sin 76^\circ} = 5.894 \dots$$

Answer: 5.89 m

Tip: You cannot find QR straight away, because you do not know the angle opposite it. Find the other two angles first, and keep them unrounded in your calculator.

Marks: 1 mark for $\sin R = \frac{5.8 \sin 76^\circ}{7.2}$. 1 mark for angle $PRQ = 51.4^\circ$. 1 mark for angle $QPR = 52.6^\circ$. 1 mark for the sine rule for QR . 1 mark for 5.89.

6 (4 marks)

The diagram shows the positions of A , B and L .

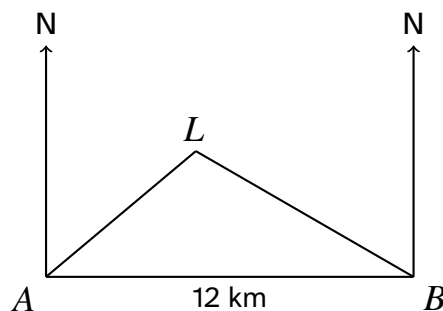


Diagram **NOT** accurately drawn

Two ships, A and B , are 12 km apart. A is due West of B .

A lighthouse L is on a bearing of 050° from A and on a bearing of 300° from B .

Work out the distance AL .

Give your answer correct to 3 significant figures.

Bearings are measured clockwise from North.

Due East is 090° , so angle $LAB = 90 - 50 = 40^\circ$

Due West is 270° , so angle $LBA = 300 - 270 = 30^\circ$

Angle $ALB = 180 - 40 - 30 = 110^\circ$ (angles in a triangle)

AL is opposite the 30° angle, and 12 km is opposite 110° .

Sine rule: $\frac{AL}{\sin 30^\circ} = \frac{12}{\sin 110^\circ}$

$$AL = \frac{12 \sin 30^\circ}{\sin 110^\circ} = 6.385 \dots$$

Answer: 6.39 km

Marks: 1 mark for the angles 40° and 30° . 1 mark for 110° . 1 mark for the sine rule. 1 mark for 6.39.

7 (4 marks)

Here is a diagram.

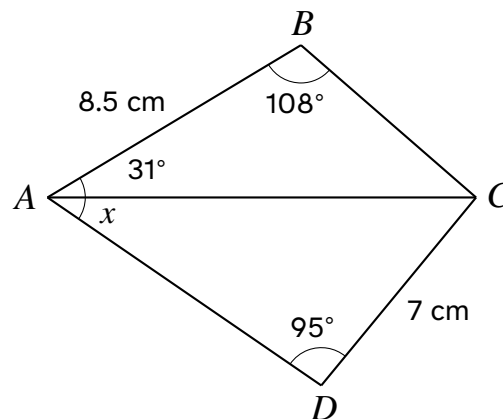


Diagram **NOT** accurately drawn

$ABCD$ is a quadrilateral.

$AB = 8.5$ cm, angle $ABC = 108^\circ$ and angle $BAC = 31^\circ$

$CD = 7$ cm and angle $ADC = 95^\circ$

Calculate the size of angle CAD , marked x .

Give your answer correct to 1 decimal place.

You must show all your working.

Step 1: find the diagonal AC , which is in both triangles.

In triangle ABC : angle $ACB = 180 - 108 - 31 = 41^\circ$

AC is opposite 108° and $AB = 8.5$ is opposite 41° .

$$AC = \frac{8.5 \sin 108^\circ}{\sin 41^\circ} = 12.32 \dots \text{ cm}$$

Step 2: use triangle ACD .

$CD = 7$ is opposite x , and AC is opposite 95° .

$$\frac{\sin x}{7} = \frac{\sin 95^\circ}{12.32 \dots}$$

$$\sin x = \frac{7 \sin 95^\circ}{12.32 \dots} = 0.5659 \dots$$

$$x = 34.46 \dots$$

Answer: 34.5°

Tip: Use the full value of AC (store it in your calculator's memory), not a rounded value.

Marks: 1 mark for angle $ACB = 41^\circ$. 1 mark for $AC = 12.3 \dots$ 1 mark for a correct sine rule for x .
1 mark for 34.5.

8 (5 marks)

The diagram shows a tower on horizontal ground.

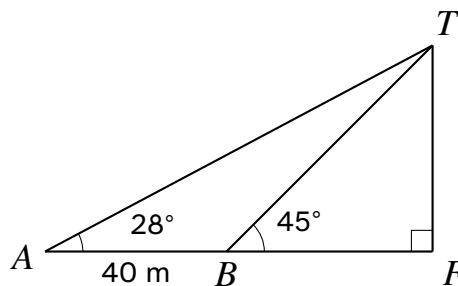


Diagram **NOT** accurately drawn

A , B and F lie on a straight line on horizontal ground. TF is a vertical tower.

Angle $TAF = 28^\circ$ and angle $TBF = 45^\circ$

$AB = 40$ m

Work out the height of the tower.

Give your answer correct to 3 significant figures.

Angles on a straight line: angle $ABT = 180 - 45 = 135^\circ$

Angles in a triangle: angle $ATB = 180 - 28 - 135 = 17^\circ$

In triangle ABT , BT is opposite 28° and $AB = 40$ is opposite 17° .

$$\frac{BT}{\sin 28^\circ} = \frac{40}{\sin 17^\circ}$$

$$BT = \frac{40 \sin 28^\circ}{\sin 17^\circ} = 64.22 \dots \text{ m}$$

Triangle BFT has a right angle at F , so use SOHCAHTOA:

$$TF = BT \sin 45^\circ = 64.22 \dots \times \sin 45^\circ = 45.41 \dots$$

Answer: 45.4 m

Tip: Triangle ABT has no right angle, so use the sine rule there. Only use SOHCAHTOA in the right-angled triangle BFT .

Marks: 1 mark for angle $ABT = 135^\circ$ or angle $ATB = 17^\circ$. 1 mark for the sine rule for BT (or AT).
1 mark for $BT = 64.2 \dots$. 1 mark for $TF = BT \sin 45^\circ$. 1 mark for 45.4.
