

The Quadratic Formula

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Solve $3x^2 + 2x - 4 = 0$

Give your solutions correct to 2 decimal places.

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 3$, $b = 2$ and $c = -4$.

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-4)}}{2 \times 3}$$

$$b^2 - 4ac = 4 + 48 = 52$$

$$\text{So } x = \frac{-2 \pm \sqrt{52}}{6}$$

Using +: $x = 0.868 \dots$

Using -: $x = -1.535 \dots$

Answer: $x = 0.87$ and $x = -1.54$

Tip: The fraction line goes under the **whole** top, $-b \pm \sqrt{b^2 - 4ac}$, not just under the square root.

Marks: 1 mark for correct substitution into the formula. 1 mark for $\frac{-2 \pm \sqrt{52}}{6}$. 1 mark for both solutions.

2 (3 marks)

Solve $5x^2 - 3x - 1 = 0$

Give your solutions correct to 3 significant figures.

Here $a = 5$, $b = -3$ and $c = -1$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-1)}}{2 \times 5}$$

$$b^2 - 4ac = 9 + 20 = 29$$

$$\text{So } x = \frac{3 \pm \sqrt{29}}{10}$$

Using +: $x = 0.8385 \dots$

Using -: $x = -0.2385 \dots$

Answer: $x = 0.839$ and $x = -0.239$

Tip: Use both + and - to get **two** solutions. Using only + loses a mark.

Marks: 1 mark for correct substitution into the formula. 1 mark for $\frac{3 \pm \sqrt{29}}{10}$. 1 mark for both solutions.

3 (3 marks)

Solve $6x^2 + 7x - 4 = 0$

Give your solutions correct to 3 significant figures.

Here $a = 6$, $b = 7$ and $c = -4$.

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 6 \times (-4)}}{2 \times 6}$$

$$b^2 - 4ac = 49 + 96 = 145$$

$$\text{So } x = \frac{-7 \pm \sqrt{145}}{12}$$

Using +: $x = 0.42013 \dots$

Using $-$: $x = -1.5868 \dots$

Answer: $x = 0.420$ and $x = -1.59$

Tip: For 3 significant figures, $0.42013 \dots$ rounds to 0.420 . The first significant figure is the 4, not the 0, and you must keep the final zero.

Marks: 1 mark for correct substitution into the formula. 1 mark for $\frac{-7 \pm \sqrt{145}}{12}$. 1 mark for both solutions.

4 (3 marks)

Solve $x(x - 2) = 5$

Give your solutions in the form $a \pm \sqrt{b}$

Expand and rearrange so that one side is 0:

$$x^2 - 2x = 5, \text{ so } x^2 - 2x - 5 = 0$$

Now $a = 1$, $b = -2$ and $c = -5$.

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{2 \pm \sqrt{24}}{2}$$

Simplify the surd: $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$

$$\text{So } x = \frac{2 \pm 2\sqrt{6}}{2}$$

Divide both terms on the top by 2: $x = 1 \pm \sqrt{6}$

Answer: $x = 1 \pm \sqrt{6}$

Marks: 1 mark for rearranging to $x^2 - 2x - 5 = 0$. 1 mark for correct substitution into the formula. 1 mark for the correct answer.

5 (3 marks)

The length of a rectangle is $(x + 5)$ cm.

The width of the rectangle is $(x - 1)$ cm.

The area of the rectangle is 35 cm^2 .

Work out the value of x .

Give your answer correct to 3 significant figures.

Area = length $\times$ width, so $(x + 5)(x - 1) = 35$

Expand: $x^2 - x + 5x - 5 = 35$, so $x^2 + 4x - 5 = 35$

Rearrange so one side is 0: $x^2 + 4x - 40 = 0$

Use the formula with $a = 1$, $b = 4$ and $c = -40$:

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-40)}}{2 \times 1} = \frac{-4 \pm \sqrt{176}}{2}$$

$$x = 4.633 \dots \text{ or } x = -8.633 \dots$$

The width $x - 1$ must be positive, so x must be more than 1.

So reject $-8.633 \dots$ and use $x = 4.633 \dots$

Answer: $x = 4.63$

Tip: Set the expression equal to 35 and then subtract 35, so that one side is 0, before you use the formula. Reject the negative solution, because a length cannot be negative.

Marks: 1 mark for $x^2 + 4x - 40 = 0$. 1 mark for correct substitution into the formula. 1 mark for 4.63 only.

6 (6 marks)

The diagram shows a right-angled triangle.
The lengths of the sides are given in centimetres.

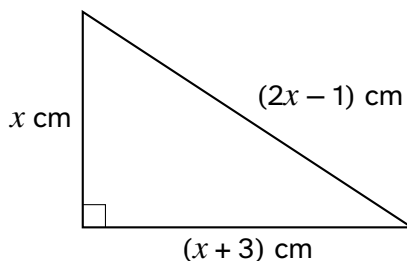


Diagram **NOT** accurately drawn

- (a) Show that $x^2 - 5x - 4 = 0$

The longest side (the hypotenuse) is $2x - 1$.

Use Pythagoras: $x^2 + (x + 3)^2 = (2x - 1)^2$

Expand: $(x + 3)^2 = x^2 + 6x + 9$ and $(2x - 1)^2 = 4x^2 - 4x + 1$

So $x^2 + x^2 + 6x + 9 = 4x^2 - 4x + 1$

$2x^2 + 6x + 9 = 4x^2 - 4x + 1$

Subtract the left side from both sides: $0 = 2x^2 - 10x - 8$

Divide every term by 2: $x^2 - 5x - 4 = 0$ as required.

Answer: Shown

Tip: $(x + 3)^2$ is not $x^2 + 9$. Write it as $(x + 3)(x + 3)$ and expand to get the middle term $6x$.

Marks: 1 mark for using Pythagoras with the three sides. 1 mark for expanding both squares correctly. 1 mark for fully correct working to $x^2 - 5x - 4 = 0$.

- (b) Work out the perimeter of the triangle.
Give your answer correct to 3 significant figures.

Solve $x^2 - 5x - 4 = 0$ with the formula ($a = 1$, $b = -5$, $c = -4$):

$$x = \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 \pm \sqrt{41}}{2}$$

$x = 5.7015 \dots$ or $x = -0.7015 \dots$

A length cannot be negative, so $x = 5.7015 \dots$

$$\begin{aligned}\text{Perimeter} &= x + (x + 3) + (2x - 1) = 4x + 2 \\ &= 4 \times 5.7015 \dots + 2 = 24.806 \dots\end{aligned}$$

Answer: 24.8 cm

Marks: 1 mark for correct substitution into the formula. 1 mark for using the positive solution in $4x + 2$. 1 mark for 24.8.

7 (5 marks)

Solve $\frac{2}{x+1} + \frac{3}{x} = 1$

Give your solutions correct to 2 decimal places.

Clear the fractions by multiplying every term by $x(x+1)$:

$$\frac{2}{x+1} \times x(x+1) = 2x$$

$$\frac{3}{x} \times x(x+1) = 3(x+1)$$

$$1 \times x(x+1) = x(x+1)$$

$$\text{So } 2x + 3(x+1) = x(x+1)$$

$$\text{Expand: } 2x + 3x + 3 = x^2 + x$$

$$5x + 3 = x^2 + x$$

$$\text{Rearrange so one side is 0: } x^2 - 4x - 3 = 0$$

Use the formula with $a = 1$, $b = -4$ and $c = -3$:

$$x = \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$x = 4.645 \dots \text{ or } x = -0.645 \dots$$

Answer: $x = 4.65$ and $x = -0.65$

Tip: Multiply the 1 on the right by $x(x + 1)$ too. Then collect everything on the side where x^2 is positive, to get a 3-term quadratic equal to 0.

Marks: 1 mark for multiplying by $x(x + 1)$. 1 mark for expanding correctly. 1 mark for $x^2 - 4x - 3 = 0$. 1 mark for correct substitution into the formula. 1 mark for both solutions.
