

The Cosine Rule

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here is a triangle.

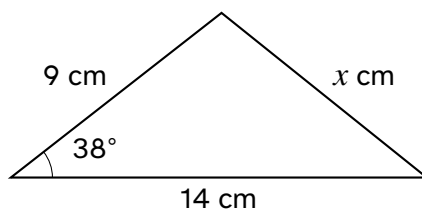


Diagram **NOT** accurately drawn

Work out the value of x .

Give your answer correct to 3 significant figures.

We know two sides (9 cm and 14 cm) and the angle between them (38°).

So use the cosine rule to find the third side, x .

$$x^2 = 9^2 + 14^2 - 2 \times 9 \times 14 \times \cos 38^\circ$$

$$x^2 = 277 - 198.5 \dots = 78.42 \dots$$

$$\text{Square root: } x = \sqrt{78.42 \dots} = 8.855 \dots$$

Answer: $x = 8.86$

Tip: Type the whole calculation in one go. A common mistake is to work out $277 - 252 = 25$ first and then multiply by $\cos 38^\circ$.

Marks: 1 mark for the correct substitution. 1 mark for the square root. 1 mark for 8.86.

2 (3 marks)

PQR is a triangle.

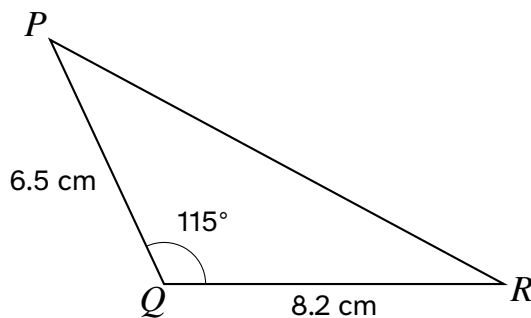


Diagram **NOT** accurately drawn

Calculate the length of PR .

Give your answer correct to 3 significant figures.

PR is opposite the 115° angle at Q .

$$PR^2 = 6.5^2 + 8.2^2 - 2 \times 6.5 \times 8.2 \times \cos 115^\circ$$

$\cos 115^\circ$ is negative, so the last term is added:

$$PR^2 = 109.49 + 45.05 \dots = 154.5 \dots$$

$$PR = \sqrt{154.5 \dots} = 12.43 \dots$$

Answer: 12.4 cm

Tip: Remember to square root at the end. 154.5 is PR^2 , not PR .

Marks: 1 mark for the correct substitution. 1 mark for the square root. 1 mark for 12.4.

3 (3 marks)

ABC is a triangle with $AB = 8$ cm, $BC = 6$ cm and $AC = 11$ cm.

Work out the size of angle ABC .

Give your answer correct to 1 decimal place.

Angle ABC is the angle at B .

The side opposite B is $AC = 11$ cm.

Use the cosine rule for an angle: $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$, where $b = AC$.

$$\cos B = \frac{8^2 + 6^2 - 11^2}{2 \times 8 \times 6} = \frac{-21}{96} = -0.21875$$

$$B = \cos^{-1}(-0.21875) = 102.63 \dots$$

Answer: 102.6°

Tip: Use the form $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ to find an angle. It is easier than rearranging the cosine rule yourself.

Marks: 1 mark for the correct substitution. 1 mark for $\cos B = -0.21875$. 1 mark for 102.6.

4 (3 marks)

A triangle has sides 4 cm, 6 cm and $2\sqrt{10}$ cm. The angle between the sides of length 4 cm and 6 cm is θ .

Show that $\cos \theta = \frac{1}{4}$

θ is between the 4 cm and 6 cm sides, so it is opposite the $2\sqrt{10}$ cm side.

$$\cos \theta = \frac{4^2 + 6^2 - (2\sqrt{10})^2}{2 \times 4 \times 6}$$

$$(2\sqrt{10})^2 = 4 \times 10 = 40$$

$$\cos \theta = \frac{16 + 36 - 40}{48} = \frac{12}{48}$$

$$\frac{12}{48} = \frac{1}{4} \quad \checkmark$$

Tip: Square both parts: $(2\sqrt{10})^2 = 2^2 \times (\sqrt{10})^2 = 4 \times 10 = 40$, not 20.

Marks: 1 mark for using the cosine rule. 1 mark for $(2\sqrt{10})^2 = 40$. 1 mark for $\frac{1}{4}$ from correct working.

5 (4 marks)

A , B and C are three buoys used in a sailing race.

B is 3.2 km due east of A .

C is 4.1 km from A and 2.5 km from B .

C is north of the line AB .

Work out the bearing of C from A .

Give your answer correct to the nearest degree.

Draw a sketch: AB points due east, with C above the line AB .

First find angle BAC . The side opposite A is $BC = 2.5$ km.

$$\cos A = \frac{4.1^2 + 3.2^2 - 2.5^2}{2 \times 4.1 \times 3.2}$$

$$\cos A = \frac{16.81 + 10.24 - 6.25}{26.24} = \frac{20.8}{26.24} = 0.7926 \dots$$

$$A = \cos^{-1}(0.7926 \dots) = 37.57 \dots^\circ$$

Due east is a bearing of 090° . C is $37.57 \dots^\circ$ north of east.

$$\text{Bearing of } C \text{ from } A = 90 - 37.57 \dots = 52.42 \dots$$

$$= 052^\circ \text{ (nearest degree)}$$

Answer: 052°

Tip: Angle BAC is not the bearing. Bearings are measured clockwise from North, so subtract angle BAC from 90° . Write a bearing with three figures.

Marks: 1 mark for a correct cosine rule for angle A . 1 mark for $\cos A = 0.79 \dots$ 1 mark for angle $A = 37.6^\circ$. 1 mark for 052° .

6 (4 marks)

Here is triangle ABC .

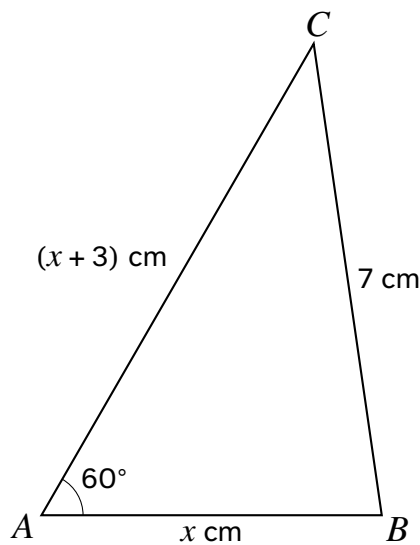


Diagram **NOT** accurately drawn

$AB = x \text{ cm}$, $AC = (x + 3) \text{ cm}$, $BC = 7 \text{ cm}$ and angle $BAC = 60^\circ$

$$\cos 60^\circ = 0.5$$

Work out the value of x .

You must show all your working.

$BC = 7$ is opposite the 60° angle.

$$7^2 = x^2 + (x + 3)^2 - 2 \times x \times (x + 3) \times 0.5$$

$$2 \times x \times (x + 3) \times 0.5 = x(x + 3) = x^2 + 3x$$

$$(x + 3)^2 = x^2 + 6x + 9$$

$$49 = x^2 + x^2 + 6x + 9 - x^2 - 3x$$

$$49 = x^2 + 3x + 9$$

$$x^2 + 3x - 40 = 0$$

$$\text{Factorise: } (x + 8)(x - 5) = 0$$

$$x = -8 \text{ or } x = 5$$

A length cannot be negative, so $x = 5$.

Answer: $x = 5$

Marks: 1 mark for a correct cosine rule equation. 1 mark for $x^2 + 3x - 40 = 0$. 1 mark for factorising (or using the formula). 1 mark for 5 only.

7 (5 marks)

$ABCD$ is a quadrilateral.

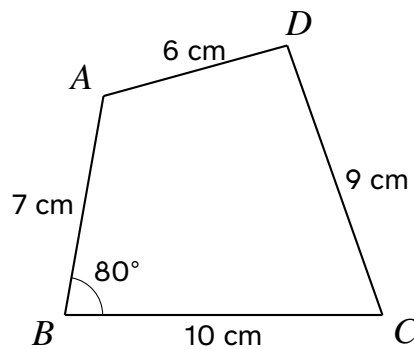


Diagram **NOT** accurately drawn

Calculate the size of angle ADC .

Give your answer correct to 1 decimal place.

You must show all your working.

Step 1: find the diagonal AC , using triangle ABC .

$$AC^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 80^\circ$$

$$AC^2 = 149 - 24.3 \dots = 124.68 \dots$$

Step 2: find angle D in triangle ACD .

AC is opposite angle D .

$$\cos D = \frac{6^2 + 9^2 - AC^2}{2 \times 6 \times 9}$$

$$\cos D = \frac{36 + 81 - 124.68 \dots}{108} = -0.0711 \dots$$

$$D = \cos^{-1}(-0.0711 \dots) = 94.08 \dots$$

Answer: 94.1°

Tip: Use the exact value of AC^2 from your calculator. Rounding AC to 11.2 changes the answer to 94.5° .

Marks: 1 mark for the cosine rule for AC . 1 mark for $AC^2 = 124.7$ or $AC = 11.2$. 1 mark for the cosine rule in triangle ACD . 1 mark for $\cos D = -0.071 \dots$. 1 mark for 94.1 .
