

Surds

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

- (a) Write $\sqrt{72}$ in the form $k\sqrt{2}$

Find the largest square number that is a factor of 72: $72 = 36 \times 2$

$$\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

Answer: $6\sqrt{2}$

- (b) Simplify $\sqrt{75} - \sqrt{12}$

$$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$$

Answer: $3\sqrt{3}$

Tip: $\sqrt{75} - \sqrt{12}$ is **not** $\sqrt{63}$. Simplify each surd first, then subtract like terms.

Marks: 1 mark for $5\sqrt{3}$ or $2\sqrt{3}$. 1 mark for $3\sqrt{3}$.

2 (4 marks)

- (a) Expand and simplify $(4 - \sqrt{3})(2 + \sqrt{3})$

Multiply each term in the first bracket by each term in the second:

$$4 \times 2 = 8$$

$$4 \times \sqrt{3} = 4\sqrt{3}$$

$$-\sqrt{3} \times 2 = -2\sqrt{3}$$

$$-\sqrt{3} \times \sqrt{3} = -3$$

$$\text{Collect like terms: } 8 - 3 + 4\sqrt{3} - 2\sqrt{3} = 5 + 2\sqrt{3}$$

Answer: $5 + 2\sqrt{3}$

Marks: 1 mark for four terms with at least three correct. 1 mark for $5 + 2\sqrt{3}$.

(b) Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$

$$\sqrt{7} \times \sqrt{7} = 7$$

$$\sqrt{7} \times (-2) = -2\sqrt{7}$$

$$2 \times \sqrt{7} = 2\sqrt{7}$$

$$2 \times (-2) = -4$$

$$\text{The } \sqrt{7} \text{ terms cancel out: } 7 - 2\sqrt{7} + 2\sqrt{7} - 4 = 3$$

Answer: 3

Marks: 1 mark for four terms with at least three correct. 1 mark for 3.

3 (2 marks)

$$\sqrt{20} + \sqrt{80} = \sqrt{n}$$

Find the value of n .

$$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

$$\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$

$$\text{Add: } 2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}$$

$$\text{Write as one surd: } 6\sqrt{5} = \sqrt{36} \times \sqrt{5} = \sqrt{180}$$

So $n = 180$

Answer: 180

Tip: $\sqrt{20} + \sqrt{80}$ is not $\sqrt{100}$. You cannot add the numbers under the roots.

Marks: 1 mark for $2\sqrt{5}$ and $4\sqrt{5}$ (or $6\sqrt{5}$). 1 mark for 180.

4 (3 marks)

A square has an area of 48 cm^2 .
Find the perimeter of the square.
Give your answer in the form $a\sqrt{3}$.

$$\text{Side length} = \sqrt{\text{area}} = \sqrt{48}$$

$$48 = 16 \times 3, \text{ so } \sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$

A square has 4 equal sides.

$$\text{Perimeter} = 4 \times 4\sqrt{3} = 16\sqrt{3}$$

Answer: $16\sqrt{3} \text{ cm}$

Tip: Use the **largest** square factor. $48 = 4 \times 12$ gives $2\sqrt{12}$, which can still be simplified.

Marks: 1 mark for $\sqrt{48}$. 1 mark for $4\sqrt{3}$. 1 mark for $16\sqrt{3}$.

5 (3 marks)

A rectangle has width $\sqrt{3} \text{ cm}$ and length $(\sqrt{50} + \sqrt{8}) \text{ cm}$.
Work out the area of the rectangle.
Give your answer in the form $a\sqrt{6}$ where a is an integer.

$$\text{Area} = \text{width} \times \text{length} = \sqrt{3} (\sqrt{50} + \sqrt{8})$$

Simplify inside the bracket first:

$$\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$$

$$\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

$$\text{So } \sqrt{50} + \sqrt{8} = 7\sqrt{2}$$

Multiply by $\sqrt{3}$: $\sqrt{3} \times 7\sqrt{2} = 7\sqrt{6}$

Answer: $7\sqrt{6} \text{ cm}^2$

Tip: $\sqrt{3} \times \sqrt{2} = \sqrt{6}$, not $\sqrt{5}$ and not 6. Multiply the numbers under the roots.

Marks: 1 mark for $5\sqrt{2}$ or $2\sqrt{2}$ (or $\sqrt{150}$ or $\sqrt{24}$). 1 mark for $7\sqrt{2}$ (or $5\sqrt{6} + 2\sqrt{6}$). 1 mark for $7\sqrt{6}$.

6 (2 marks)

Is $(\sqrt{12} + \sqrt{3})^2$ rational or irrational?

You must show your working.

Simplify inside the bracket: $\sqrt{12} = 2\sqrt{3}$

So $\sqrt{12} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

Square it: $(3\sqrt{3})^2 = 3^2 \times (\sqrt{3})^2 = 9 \times 3 = 27$

27 is a whole number, so it is rational.

Answer: Rational (27)

Marks: 1 mark for a correct method that reaches 27. 1 mark for “rational” with 27 shown.

7 (3 marks)

Show that $(2\sqrt{5} - 1)^2 + 3(1 - \sqrt{5})$ can be written in the form $a - b\sqrt{5}$, where a and b are integers.

Expand the squared bracket: $(2\sqrt{5} - 1)(2\sqrt{5} - 1)$

$2\sqrt{5} \times 2\sqrt{5} = 4 \times 5 = 20$

Middle terms: $-2\sqrt{5} - 2\sqrt{5} = -4\sqrt{5}$

$(-1) \times (-1) = +1$

So $(2\sqrt{5} - 1)^2 = 21 - 4\sqrt{5}$

Expand the other bracket: $3(1 - \sqrt{5}) = 3 - 3\sqrt{5}$

Add: $21 - 4\sqrt{5} + 3 - 3\sqrt{5} = 24 - 7\sqrt{5}$

This is in the form $a - b\sqrt{5}$ with $a = 24$ and $b = 7$ ✓

Answer: $24 - 7\sqrt{5}$

Tip: $2\sqrt{5} \times 2\sqrt{5} = 4 \times 5 = 20$, not 10 and not $4\sqrt{5}$.

Marks: 1 mark for expanding $(2\sqrt{5} - 1)^2$ with at least three of the four terms correct. 1 mark for $21 - 4\sqrt{5}$. 1 mark for $24 - 7\sqrt{5}$.

8 (3 marks)

The diagram shows a right-angled triangle.

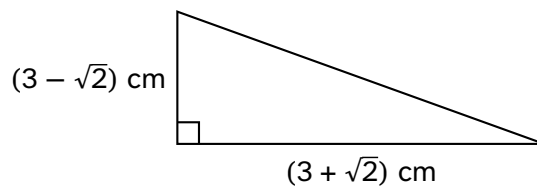


Diagram **NOT** accurately drawn

Show that the length of the hypotenuse of the triangle is $\sqrt{22}$ cm.
You must show all your working.

Use Pythagoras: $\text{hypotenuse}^2 = (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2$

$$(3 + \sqrt{2})^2 = 9 + 6\sqrt{2} + 2 = 11 + 6\sqrt{2}$$

$$(3 - \sqrt{2})^2 = 9 - 6\sqrt{2} + 2 = 11 - 6\sqrt{2}$$

$$\text{Add: } (11 + 6\sqrt{2}) + (11 - 6\sqrt{2}) = 22$$

So $\text{hypotenuse}^2 = 22$ and the hypotenuse is $\sqrt{22}$ cm ✓

Answer: $(11 + 6\sqrt{2}) + (11 - 6\sqrt{2}) = 22$

Marks: 1 mark for one correct expansion. 1 mark for adding the two expansions. 1 mark for 22 and a conclusion.
