

Spheres, Cones, Pyramids and Frustums

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Here is a square-based pyramid.

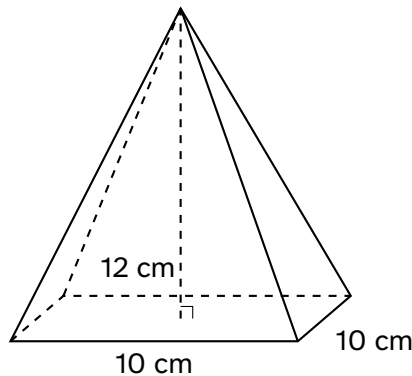


Diagram **NOT** accurately drawn

The base is a square of side 10 cm.

The perpendicular height of the pyramid is 12 cm.

Work out the volume of the pyramid.

Volume of a pyramid = $\frac{1}{3} \times$ area of base $\times$ height

Area of base = $10 \times 10 = 100 \text{ cm}^2$

Volume = $\frac{1}{3} \times 100 \times 12 = 400$

Answer: 400 cm³

Marks: 1 mark for $\frac{1}{3} \times 100 \times 12$. 1 mark for 400.

2 (3 marks)

A hemisphere has a total surface area (including the flat face) of $75\pi \text{ cm}^2$.
Find its radius.

$$\text{Curved surface} = \frac{1}{2} \times 4\pi r^2 = 2\pi r^2$$

$$\text{Flat circular face} = \pi r^2$$

$$\text{Total} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

$$3\pi r^2 = 75\pi$$

$$\text{Divide both sides by } 3\pi: r^2 = 25$$

$$r = 5$$

Answer: 5 cm

Tip: The curved part of a hemisphere is **half** of $4\pi r^2$. Don't forget to add the flat circle, πr^2 .

Marks: 1 mark for $2\pi r^2$ (half of $4\pi r^2$). 1 mark for $3\pi r^2 = 75\pi$. 1 mark for 5.

3 (3 marks)

A solid metal sphere of radius 6 cm is melted down and made into a cone of radius 8 cm.

Work out the height of the cone.

Melting does not change the volume, so the cone has the same volume as the sphere.

$$\text{Volume of sphere} = \frac{4}{3}\pi \times 6^3 = \frac{4}{3}\pi \times 216 = 288\pi$$

$$\text{Volume of cone} = \frac{1}{3}\pi \times 8^2 \times h = \frac{64}{3}\pi h$$

$$\frac{64}{3}\pi h = 288\pi$$

$$h = 288 \times \frac{3}{64} = 13.5$$

Answer: 13.5 cm

Tip: In the sphere formula, cube the radius: $6^3 = 216$, not 6^2 .

Marks: 1 mark for 288π . 1 mark for $\frac{1}{3}\pi \times 64 \times h = 288\pi$. 1 mark for 13.5.

4 (3 marks)

The diagram shows a frustum.

It is made by removing a cone of radius 4 cm and height 6 cm from the top of a cone of radius 10 cm and height 15 cm.

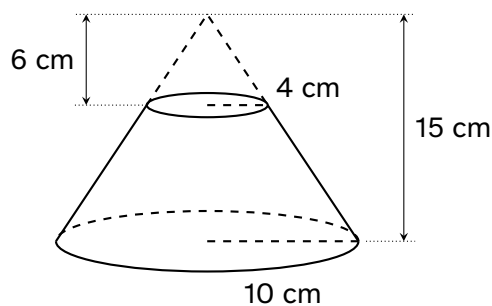


Diagram **NOT** accurately drawn

Work out the volume of the frustum.

Give your answer correct to 3 significant figures.

Frustum = large cone – small cone

$$\text{Large cone: } \frac{1}{3}\pi \times 10^2 \times 15 = 500\pi$$

$$\text{Small cone: } \frac{1}{3}\pi \times 4^2 \times 6 = 32\pi$$

$$\text{Frustum} = 500\pi - 32\pi = 468\pi = 1470.2\dots$$

$$= 1470 \text{ (3 s.f.)}$$

Answer: 1470 cm^3

Tip: You cannot put the frustum's own height into the cone formula. Find the two cone volumes and subtract.

Marks: 1 mark for the volume of either cone. 1 mark for subtracting. 1 mark for 1470.

5 (2 marks)

A sphere has radius r cm.

A cylinder has radius r cm and height h cm.

The surface area of the sphere is equal to the area of the curved surface of the cylinder.
Work out the ratio $h:r$

$$\text{Surface area of the sphere} = 4\pi r^2$$

$$\text{Curved surface of the cylinder} = 2\pi r \times h = 2\pi rh$$

$$4\pi r^2 = 2\pi rh$$

$$\text{Divide both sides by } 2\pi r: 2r = h$$

$$h:r = 2r:r = 2:1$$

Answer: $2:1$

Tip: The curved surface of a cylinder is a rectangle when opened out: circumference $\times$ height $= 2\pi rh$. Check the order: the question asks for $h:r$.

Marks: 1 mark for $4\pi r^2 = 2\pi rh$. 1 mark for $2:1$.

6 (4 marks)

The diagram shows a solid made from a hemisphere on top of a cylinder.
The hemisphere and the cylinder each have radius r cm.
The height of the cylinder is 12 cm.
The volume of the hemisphere is $\frac{1}{3}$ of the volume of the cylinder.

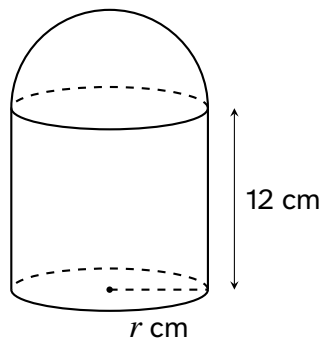


Diagram **NOT** accurately drawn

Work out the total volume of the solid.

Give your answer in terms of π .

You must show all your working.

$$\text{Volume of the hemisphere} = \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{2}{3} \pi r^3$$

$$\text{Volume of the cylinder} = \pi r^2 \times 12 = 12\pi r^2$$

$$\text{The hemisphere is } \frac{1}{3} \text{ of the cylinder: } \frac{2}{3} \pi r^3 = \frac{1}{3} \times 12\pi r^2 = 4\pi r^2$$

$$\text{Divide both sides by } \frac{2}{3} \pi r^2: r = 6$$

$$\text{Cylinder: } 12\pi \times 6^2 = 432\pi$$

$$\text{Hemisphere: } \frac{2}{3} \pi \times 6^3 = 144\pi$$

$$\text{Total} = 432\pi + 144\pi = 576\pi$$

Answer: $576\pi \text{ cm}^3$

Tip: A hemisphere is half a sphere, so its volume is $\frac{2}{3} \pi r^3$. Form an equation first, then solve it for r .

Marks: 1 mark for $\frac{2}{3} \pi r^3 = \frac{1}{3} \times 12\pi r^2$ (or equivalent). 1 mark for $r = 6$. 1 mark for 432π or 144π . 1 mark for 576π .

7 (4 marks)

The diagram shows a lampshade.

The lampshade is the curved surface of a frustum of a cone.

The top has radius 6 cm and the bottom has radius 15 cm.

The sloping edge of the lampshade is 15 cm long.

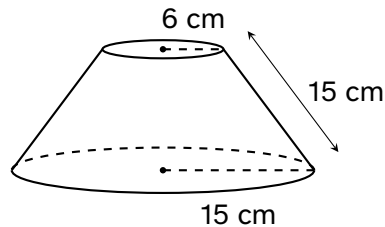


Diagram **NOT** accurately drawn

Work out the area of the curved surface of the lampshade.

Give your answer correct to 3 significant figures.

Think of the lampshade as a large cone with a small cone removed from the top.

Let the slant height of the large cone be L cm. The small cone's slant height is $L - 15$.

The cones are similar, so $\frac{L - 15}{L} = \frac{6}{15}$

$$15(L - 15) = 6L, \text{ so } 15L - 225 = 6L, 9L = 225, L = 25$$

$$\text{Slant height of the small cone} = 25 - 15 = 10 \text{ cm}$$

$$\text{Curved surface} = \pi \times 15 \times 25 - \pi \times 6 \times 10 = 375\pi - 60\pi = 315\pi$$

$$315\pi = 989.6 \dots$$

$$= 990 \text{ (3 s.f.)}$$

Answer: 990 cm²

Tip: You cannot use $\pi r l$ with $l = 15$ for a frustum. Complete the cone, find both slant heights, then subtract the two curved surfaces.

Marks: 1 mark for using similar cones (e.g. $\frac{L-15}{L} = \frac{6}{15}$). 1 mark for slant heights 25 and 10. 1 mark for $375\pi - 60\pi$. 1 mark for 990.

8 (5 marks)

The diagram shows a solid toy made from a cone and a hemisphere.
The cone and the hemisphere both have radius 5 cm.
The vertical height of the cone is 12 cm.

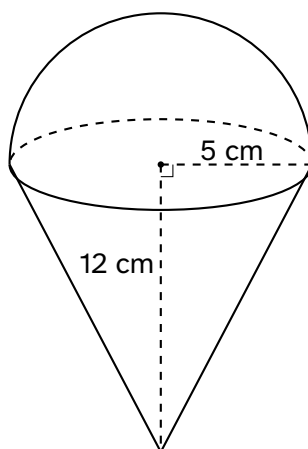


Diagram **NOT** accurately drawn

Calculate the total surface area of the toy.
Give your answer correct to 3 significant figures.

The surface is the curved surface of the cone plus the curved surface of the hemisphere.

$$\text{Slant height of the cone: } l = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ cm}$$

$$\text{Curved surface of the cone} = \pi r l = \pi \times 5 \times 13 = 65\pi$$

$$\text{Curved surface of the hemisphere} = \frac{1}{2} \times 4\pi \times 5^2 = 50\pi$$

There is no flat circle on the outside, because the cone and hemisphere are joined there.

$$\text{Total} = 65\pi + 50\pi = 115\pi = 361.2 \dots$$

$$= 361 \text{ (3 s.f.)}$$

Answer: 361 cm^2

Tip: Use the slant height (13), not the vertical height (12), in $\pi r l$. A hemisphere's curved surface is half of $4\pi r^2$. Do not include the circle where the two parts are joined.

Marks: 1 mark for $l = 13$. 1 mark for 65π . 1 mark for 50π . 1 mark for $65\pi + 50\pi$. 1 mark for 361.