

Simplifying Algebraic Fractions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (5 marks)

Simplify fully

(a) $\frac{x+3}{4(x+3)^2}$

The bottom contains $(x+3)^2 = (x+3)(x+3)$.

Cancel one factor of $(x+3)$ from the top and the bottom.

This leaves 1 on the top.

Answer: $\frac{1}{4(x+3)}$

(b) $\frac{10x^3y^2}{15x^5y}$

Simplify the numbers: $\frac{10}{15} = \frac{2}{3}$

Simplify the x terms: $\frac{x^3}{x^5} = \frac{1}{x^2}$

Simplify the y terms: $\frac{y^2}{y} = y$

Put these together.

Answer: $\frac{2y}{3x^2}$

Marks: 2 marks for the correct answer. 1 mark if two of the three parts (number, x , y) are correct.

(c) $\frac{6x + 18}{x^2 - 9}$

Factorise the top: $6x + 18 = 6(x + 3)$

Factorise the bottom (difference of two squares): $x^2 - 9 = (x + 3)(x - 3)$

So the fraction is $\frac{6(x + 3)}{(x + 3)(x - 3)}$

Cancel the common factor $(x + 3)$.

Answer: $\frac{6}{x - 3}$

Tip: You can only cancel factors (whole brackets that multiply), not single terms.
Factorise first, then cancel.

Marks: 1 mark for factorising the top and the bottom. 1 mark for $\frac{6}{x - 3}$.

2 (3 marks)

Simplify fully $\frac{x^2 - 16}{x^2 + 2x - 8}$

Top: this is a difference of two squares.

$$x^2 - 16 = (x + 4)(x - 4)$$

Bottom: $x^2 + 2x - 8 = (x + 4)(x - 2)$

So the fraction is $\frac{(x + 4)(x - 4)}{(x + 4)(x - 2)}$

Cancel the common factor $(x + 4)$.

Answer: $\frac{x - 4}{x - 2}$

Tip: Don't cancel the x^2 terms. Factorise the top and the bottom first, then cancel a whole bracket.

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{x - 4}{x - 2}$.

3 (3 marks)

Simplify fully $\frac{x^2 + 5x}{2x^2 + 7x - 15}$

Top: take out the common factor x .

$$x^2 + 5x = x(x + 5)$$

$$\text{Bottom: } 2x^2 + 7x - 15 = (2x - 3)(x + 5)$$

So the fraction is $\frac{x(x + 5)}{(2x - 3)(x + 5)}$

Cancel the common factor $(x + 5)$.

Answer: $\frac{x}{2x - 3}$

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{x}{2x - 3}$.

4 (3 marks)

Simplify fully $\frac{3x^2 - 7x - 6}{x^2 - 9}$

$$\text{Top: } 3x^2 - 7x - 6 = (3x + 2)(x - 3)$$

$$\text{Check: } (3x + 2)(x - 3) = 3x^2 - 9x + 2x - 6 = 3x^2 - 7x - 6 \quad \checkmark$$

$$\text{Bottom (difference of two squares): } x^2 - 9 = (x + 3)(x - 3)$$

So the fraction is $\frac{(3x + 2)(x - 3)}{(x + 3)(x - 3)}$

Cancel the common factor $(x - 3)$.

Answer: $\frac{3x + 2}{x + 3}$

Tip: In these questions a bracket almost always cancels. If none does, check your factorising for a sign error.

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{3x + 2}{x + 3}$.

5 (3 marks)

Simplify fully $\frac{2x^2 - 7x - 4}{3x^2 - 12x}$

Top: $2x^2 - 7x - 4 = (2x + 1)(x - 4)$

Bottom: take out the common factor $3x$.

$$3x^2 - 12x = 3x(x - 4)$$

So the fraction is $\frac{(2x + 1)(x - 4)}{3x(x - 4)}$

Cancel the common factor $(x - 4)$.

Answer: $\frac{2x + 1}{3x}$

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{2x + 1}{3x}$.

6 (3 marks)

Write $\frac{25x^2 - 1}{10x^2 + 2x}$ in the form $\frac{ax + b}{cx}$ where a , b and c are integers.

Top: this is a difference of two squares, because $25x^2 = (5x)^2$ and $1 = 1^2$.

$$25x^2 - 1 = (5x + 1)(5x - 1)$$

Bottom: take out the common factor $2x$.

$$10x^2 + 2x = 2x(5x + 1)$$

So the fraction is $\frac{(5x + 1)(5x - 1)}{2x(5x + 1)}$

Cancel the common factor $(5x + 1)$.

This is in the form $\frac{ax+b}{cx}$, with $a = 5$, $b = -1$ and $c = 2$.

Answer: $\frac{5x-1}{2x}$

Tip: Take out the **highest** common factor from the bottom, $2x$, so that the bracket $(5x+1)$ matches the top.

Marks: 1 mark for $(5x+1)(5x-1)$. 1 mark for $2x(5x+1)$. 1 mark for $\frac{5x-1}{2x}$.

7 (3 marks)

Show that $\frac{x^2 - 2x - 15}{2x^2 + 7x + 3}$ can be written in the form $\frac{ax+b}{cx+d}$ where a , b , c and d are integers.

Top: $x^2 - 2x - 15 = (x - 5)(x + 3)$

Bottom: $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

So the fraction is $\frac{(x-5)(x+3)}{(2x+1)(x+3)}$

Cancel the common factor $(x + 3)$:

$$\frac{x-5}{2x+1}$$

This is in the form $\frac{ax+b}{cx+d}$, with $a = 1$, $b = -5$, $c = 2$ and $d = 1$.

Answer: $\frac{x-5}{2x+1}$

Marks: 1 mark for factorising the top. 1 mark for factorising the bottom. 1 mark for $\frac{x-5}{2x+1}$.

8 (3 marks)

Simplify fully $\frac{16 - x^2}{x^2 - 3x - 4}$

Top: this is a difference of two squares.

$$16 - x^2 = (4 - x)(4 + x)$$

$$\text{Bottom: } x^2 - 3x - 4 = (x - 4)(x + 1)$$

$(4 - x)$ and $(x - 4)$ are not the same, but $4 - x = -(x - 4)$.

So the top is $-(x - 4)(x + 4)$

$$\text{The fraction is } \frac{-(x - 4)(x + 4)}{(x - 4)(x + 1)}$$

Cancel the common factor $(x - 4)$.

$$\text{Answer: } -\frac{x + 4}{x + 1}$$

Tip: $16 - x^2$ factorises as $(4 - x)(4 + x)$, not $(x + 4)(x - 4)$. Then use $4 - x = -(x - 4)$ so that the brackets cancel.

Marks: 1 mark for $(4 - x)(4 + x)$. 1 mark for $(x - 4)(x + 1)$. 1 mark for the correct answer, or an equivalent such as $\frac{-x - 4}{x + 1}$.
