

Similar Shapes - Areas and Volumes

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The ratio of the volumes of two similar spheres is 8:125
Find the ratio of their surface areas.

Cube root the volume ratio to get the length ratio.

$$\text{Length ratio} = \sqrt[3]{8}:\sqrt[3]{125} = 2:5$$

Square the length ratio to get the area ratio.

$$\text{Area ratio} = 2^2:5^2 = 4:25$$

Answer: 4:25

Marks: 1 mark for the length ratio 2:5. 1 mark for 4:25.

2 (3 marks)

A garden centre sells two plant pots that are mathematically similar.
The smaller pot holds 2.5 litres of compost and is 24 cm tall.
The larger pot holds 20 litres of compost.
Work out the height of the larger pot.

$$\text{Volume scale factor} = 20 \div 2.5 = 8$$

Height is a length, so cube root:

$$\text{Length scale factor} = \sqrt[3]{8} = 2$$

Height of the larger pot = $24 \times 2 = 48$ cm

Answer: 48 cm

Tip: Do not multiply 24 by 8. The capacity is a volume, so cube root the volume scale factor to get the length scale factor.

Marks: 1 mark for $20 \div 2.5$ or 8. 1 mark for $\sqrt[3]{8} = 2$. 1 mark for 48.

3 (3 marks)

Shapes A and B are mathematically similar.

The area of A is 20 cm^2 and the area of B is 45 cm^2 .

A length on shape A is 6 cm.

Work out the corresponding length on shape B .

Area scale factor = $45 \div 20 = 2.25$

Square root to get the length scale factor:

$$\sqrt{2.25} = 1.5$$

$$\text{Length on } B = 6 \times 1.5 = 9$$

Answer: 9 cm

Marks: 1 mark for $45 \div 20$. 1 mark for the square root. 1 mark for 9.

4 (3 marks)

A model of a statue is made to a scale of 1:20

The model has a volume of 450 cm^3 .

Work out the volume of the statue.

Give your answer in m^3 .

Length scale factor = 20

Volume scale factor = $20^3 = 8000$

Volume of statue = $450 \times 8000 = 3\,600\,000 \text{ cm}^3$

$$1 \text{ m}^3 = 100 \times 100 \times 100 = 1\,000\,000 \text{ cm}^3$$

$$3\,600\,000 \div 1\,000\,000 = 3.6 \text{ m}^3$$

Answer: 3.6 m^3

Tip: To change cm^3 to m^3 , divide by $100^3 = 1\,000\,000$, not by 100.

Marks: 1 mark for 20^3 . 1 mark for dividing by 1 000 000. 1 mark for 3.6.

5 (3 marks)

A small model boat and a large model boat are mathematically similar.

The sail of the small boat has an area of 150 cm^2

The sail of the large boat has an area of 240 cm^2

The hull of the small boat has a volume of 500 cm^3

Work out the volume of the hull of the large boat.

Give your answer correct to 3 significant figures.

$$\text{Area scale factor} = 240 \div 150 = 1.6$$

$$\text{Length scale factor} = \sqrt{1.6} = 1.2649 \dots$$

$$\text{Volume scale factor} = (\sqrt{1.6})^3 = 2.0238 \dots$$

$$\text{Volume of the large hull} = 500 \times 2.0238 \dots = 1011.9 \dots$$

Round to 3 significant figures: 1010

Answer: 1010 cm^3

Tip: Keep the full calculator value of $\sqrt{1.6}$. Rounding it early can make your final answer wrong.

$500 \times 1.6 = 800$ uses the area scale factor, which is wrong for a volume.

Marks: 1 mark for $\sqrt{1.6}$ (or equivalent). 1 mark for $500 \times (\sqrt{1.6})^3$. 1 mark for 1010.

6 (3 marks)

Two glass vases, R and S , are mathematically similar.

Vase R holds 1.6 litres of water.

Vase S holds 5.4 litres of water.

The outside surface area of vase R is 480 cm^2

Show that the outside surface area of vase S is 1080 cm^2

$$\text{Volume scale factor} = 5.4 \div 1.6 = 3.375$$

$$\text{Length scale factor} = \sqrt[3]{3.375} = 1.5$$

$$\text{Area scale factor} = 1.5^2 = 2.25$$

$$\text{Surface area of } S = 480 \times 2.25 = 1080 \text{ cm}^2 \quad \checkmark$$

Tip: Do not multiply 480 by 3.375. Cube root the volume scale factor to get the length scale factor, then square it.

Marks: 1 mark for $\sqrt[3]{3.375}$ or 1.5. 1 mark for 1.5^2 or 2.25. 1 mark for getting 1080 from correct working.

7 (5 marks)

Two watering cans are mathematically similar.

The smaller can holds 2 litres of water.

The larger can holds 5 litres of water and is 40 cm tall.

- (a) Work out the height of the smaller watering can.
Give your answer correct to 1 decimal place.

$$\text{Volume scale factor from large to small} = 2 \div 5 = 0.4$$

$$\text{Length scale factor} = \sqrt[3]{0.4} = 0.7368 \dots$$

$$\text{Height of the smaller can} = 40 \times 0.7368 \dots = 29.47 \dots$$

$$\text{Round to 1 decimal place: } 29.5 \text{ cm}$$

Answer: 29.5 cm

Tip: Do not multiply 40 by 0.4. The capacities give a volume scale factor, so cube root it first.

Marks: 1 mark for $2 \div 5$ (or $5 \div 2$). 1 mark for $\sqrt[3]{0.4}$ (or $\sqrt[3]{2.5}$). 1 mark for 29.5.

- (b) The smaller watering can is made from 1800 cm^2 of metal sheet.
Work out the area of metal sheet needed to make the larger watering can.
Give your answer correct to 3 significant figures.

Length scale factor from small to large = $\sqrt[3]{2.5} = 1.3572 \dots$

Area scale factor = $(\sqrt[3]{2.5})^2 = 1.8420 \dots$

Area = $1800 \times 1.8420 \dots = 3315.6 \dots$

Round to 3 significant figures: 3320 cm^2

Answer: 3320 cm^2

Tip: The metal sheet is an area, so square the length scale factor. Do not use the volume scale factor 2.5.

Marks: 1 mark for $1800 \times (\sqrt[3]{2.5})^2$. 1 mark for 3320.

8 (4 marks)

A shop sells cat food in two sizes of tin.
The tins are mathematically similar.
The small tin is 6 cm tall and costs £0.80
The large tin is 9 cm tall and costs £2.50
Which size of tin is the better value for money?
You must show all your working.

Length scale factor = $9 \div 6 = 1.5$

Volume scale factor = $1.5^3 = 3.375$

So the large tin holds 3.375 times as much food as the small tin.

The same amount of food in small tins would cost $3.375 \times £0.80 = £2.70$

The large tin costs £2.50, which is less than £2.70.

So the large tin is the better value.

Answer: The large tin (the same amount in small tins would cost £2.70)

Tip: The large tin is 1.5 times as tall but holds $1.5^3 = 3.375$ times as much. Comparing $1.5 \times £0.80 = £1.20$ with £2.50 gives the wrong answer.

Marks: 1 mark for 1.5. 1 mark for $1.5^3 = 3.375$. 1 mark for a correct comparison, e.g. £2.70 and £2.50. 1 mark for the large tin with correct working.

9 (4 marks)

A company makes three sizes of gift box, S , M and L .

The boxes are mathematically similar.

height of S : height of $M = 2:3$

volume of M : volume of $L = 27:125$

The surface area of box S is 32 cm^2

Work out the surface area of box L .

Cube root the volume ratio: heights $M:L = \sqrt[3]{27}:\sqrt[3]{125} = 3:5$

Heights $S:M = 2:3$

The M values match (both 3), so heights $S:M:L = 2:3:5$

So heights $S:L = 2:5$

Square for areas: surface areas $S:L = 4:25$

Surface area of $L = 32 \times \frac{25}{4} = 200 \text{ cm}^2$

Answer: 200 cm^2

Tip: Turn both ratios into length ratios before you combine them. You cannot join a length ratio and a volume ratio directly.

Marks: 1 mark for 3:5. 1 mark for 2:5 (heights of S and L). 1 mark for 4:25. 1 mark for 200.
