

Recurring Decimals to Fractions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

- (a) Write $\frac{7}{11}$ as a recurring decimal.

Divide 7 by 11 (short division):

$$7 \div 11 = 0.636363 \dots$$

The block 63 repeats, so put a dot over the 6 and the 3.

Answer: $0.\dot{6}\dot{3}$

- (b) Write $0.\dot{7}$ as a fraction.

$$\text{Let } x = 0.777 \dots$$

One digit repeats, so multiply by 10: $10x = 7.777 \dots$

$$\text{Subtract: } 10x - x = 7.777 \dots - 0.777 \dots$$

$$9x = 7, \text{ so } x = \frac{7}{9}$$

Answer: $\frac{7}{9}$

2 (2 marks)

Prove algebraically that $0.\dot{3}\dot{8} = \frac{38}{99}$

$$\text{Let } x = 0.383838 \dots$$

The repeating block (38) has 2 digits, so multiply by 100:

$$100x = 38.383838 \dots$$

Subtract to remove the recurring part:

$$100x - x = 38.3838 \dots - 0.3838 \dots$$

$$99x = 38$$

$$x = \frac{38}{99} \quad \checkmark$$

Answer: $99x = 38$, so $x = \frac{38}{99}$

Tip: Show that the decimals recur, using dots or "...". Writing $100x = 38.38$ (a decimal that stops) loses marks in a proof.

Marks: 1 mark for $100x = 38.3838 \dots$ and subtracting x . 1 mark for $99x = 38$ leading to $\frac{38}{99}$.

3 (3 marks)

Kwame says,

" $0.\dot{7}\dot{2}$ is equal to $\frac{8}{11}$ "

(a) Show that Kwame is wrong.

Kwame's fraction: $\frac{8}{11} = 8 \div 11 = 0.727272 \dots = 0.\dot{7}\dot{2}$ (the 7 and the 2 both repeat)

But $0.7\dot{2} = 0.72222 \dots$ (only the 2 repeats)

$0.7272 \dots \neq 0.7222 \dots$, so Kwame is wrong.

Answer: $\frac{8}{11} = 0.7272 \dots$ but $0.7\dot{2} = 0.7222 \dots$

(b) Write $0.7\dot{2}$ as a fraction in its simplest form.

Let $x = 0.7222 \dots$

$$10x = 7.222 \dots$$

$$100x = 72.222 \dots$$

$$\text{Subtract: } 100x - 10x = 72.222 \dots - 7.222 \dots$$

$$90x = 65$$

$$x = \frac{65}{90}$$

Simplify (divide top and bottom by 5): $x = \frac{13}{18}$

Answer: $\frac{13}{18}$

Tip: Read the dots carefully. A dot over one digit means only that digit repeats; Kwame treated $0.7\dot{2}$ as if it were $0.\dot{7}\dot{2}$.

Marks: 1 mark for $90x = 65$ (or $\frac{65}{90}$). 1 mark for $\frac{13}{18}$.

4 (2 marks)

Express $0.\dot{4}\dot{0}\dot{5}$ as a fraction in its simplest form.

Let $x = 0.405405405 \dots$ (the block 405 repeats)

The block has 3 digits, so multiply by 1000:

$$1000x = 405.405405 \dots$$

$$\text{Subtract: } 1000x - x = 405$$

$$999x = 405, \text{ so } x = \frac{405}{999}$$

Simplify (divide top and bottom by 27): $x = \frac{15}{37}$

Answer: $\frac{15}{37}$

Tip: Dots over the first and last digit mean the whole block repeats: $0.\dot{4}\dot{0}\dot{5} = 0.405405 \dots$
Write out a few digits before you start.

Marks: 1 mark for $999x = 405$ (or $\frac{405}{999}$). 1 mark for $\frac{15}{37}$.

5 (3 marks)

Prove algebraically that $0.5\dot{2} = \frac{47}{90}$

Let $x = 0.5222 \dots$ (only the 2 repeats)

$$10x = 5.222 \dots$$

$$100x = 52.222 \dots$$

These have the same digits after the decimal point, so subtract:

$$100x - 10x = 52.222 \dots - 5.222 \dots$$

$$90x = 47$$

$$x = \frac{47}{90} \quad \checkmark$$

Answer: $90x = 47$, so $x = \frac{47}{90}$

Marks: 1 mark for two correct recurring decimals, e.g. $10x = 5.222 \dots$ and $100x = 52.222 \dots$ 1 mark for subtracting to get $90x = 47$. 1 mark for completing the proof.

6 (3 marks)

Express $1.2\dot{1}8$ as a mixed number in its simplest form.

Leave the whole number 1 to the end. Work with $x = 0.21818 \dots$

$$10x = 2.1818 \dots$$

$$1000x = 218.1818 \dots$$

$$\text{Subtract: } 1000x - 10x = 218.1818 \dots - 2.1818 \dots$$

$$990x = 216$$

$$x = \frac{216}{990}$$

$$\text{Simplify (divide top and bottom by 18): } x = \frac{12}{55}$$

Add the 1 back on: $1\frac{12}{55}$

Answer: $1\frac{12}{55}$

Tip: $1000x - 10x = 990x$, not $900x$. Check your subtraction of the multiples of x .

Marks: 1 mark for two correct recurring decimals that differ by a whole number. 1 mark for $\frac{216}{990}$ (or equivalent). 1 mark for $1\frac{12}{55}$.

7 (3 marks)

Prove algebraically that $0.1\dot{2}4\dot{3}$ can be written as $\frac{23}{185}$

Let $x = 0.1243243 \dots$ (the block 243 repeats)

$$10x = 1.243243 \dots$$

$$10\,000x = 1243.243243 \dots$$

These have the same digits after the decimal point, so subtract:

$$10\,000x - 10x = 1242$$

$$9990x = 1242$$

$$x = \frac{1242}{9990}$$

Simplify (divide top and bottom by 54): $x = \frac{23}{185}$ ✓

Answer: $9990x = 1242$, $\frac{1242}{9990} = \frac{23}{185}$

Marks: 1 mark for two correct recurring decimals that differ by a whole number. 1 mark for $\frac{1242}{9990}$ (or equivalent). 1 mark for completing the proof.

8 (5 marks)

Write the ratio $0.\dot{3}\dot{6} : 0.3\dot{8}$ in the form $m : n$, where m and n are integers.
You must show your working.

First write each recurring decimal as a fraction.

Let $x = 0.3636 \dots$, so $100x = 36.3636 \dots$

$$100x - x = 36, \text{ so } 99x = 36 \text{ and } x = \frac{36}{99} = \frac{4}{11}$$

Let $y = 0.3888 \dots$, so $10y = 3.888 \dots$ and $100y = 38.888 \dots$

$$100y - 10y = 35, \text{ so } 90y = 35 \text{ and } y = \frac{35}{90} = \frac{7}{18}$$

So the ratio is $\frac{4}{11} : \frac{7}{18}$

Multiply both sides by $11 \times 18 = 198$ to clear the fractions:

$$\frac{4}{11} \times 198 = 72 \text{ and } \frac{7}{18} \times 198 = 77$$

72 and 77 have no common factor, so the ratio is 72:77

Answer: 72:77

Tip: $0.\dot{3}6$ and $0.3\dot{8}$ recur in different ways: $0.3636 \dots$ and $0.3888 \dots$. Write out a few digits of each before choosing what to multiply by.

Marks: 1 mark for a correct method for one decimal (e.g. $99x = 36$). 1 mark for $\frac{4}{11}$ (or $\frac{36}{99}$). 1 mark for $\frac{7}{18}$ (or $\frac{35}{90}$). 1 mark for multiplying both fractions by a common multiple of the denominators. 1 mark for 72:77.

9 (2 marks)

x is an integer such that $\frac{x}{11}$ is between $0.\dot{3}$ and $0.\dot{5}$

Find all the possible values of x .

You must show your working.

Write the decimals as fractions:

$$0.\dot{3} = \frac{1}{3} \text{ and } 0.\dot{5} = \frac{5}{9}$$

$$\text{So } \frac{1}{3} < \frac{x}{11} < \frac{5}{9}$$

Multiply everything by 11:

$$11 \times \frac{1}{3} = 3.66 \dots \text{ and } 11 \times \frac{5}{9} = 6.11 \dots$$

So x is between $3.66 \dots$ and $6.11 \dots$

The integers in this range are 4, 5 and 6.

Answer: 4, 5, 6

Marks: 1 mark for $\frac{1}{3}$ and $\frac{5}{9}$ (or $3.66 \dots$ and $6.11 \dots$). 1 mark for 4, 5, 6.
