

Recognising and Sketching Graphs

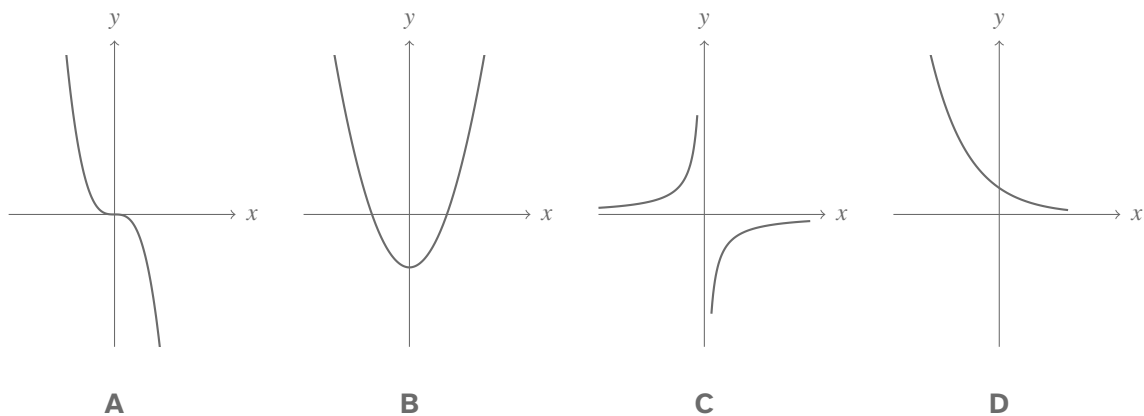
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here are four graphs.



Match each equation to its graph.

$$y = x^2 - 2 \quad y = -x^3 \quad y = \left(\frac{1}{2}\right)^x \quad y = -\frac{1}{x}$$

$y = x^2 - 2$ is a quadratic (U-shape) crossing the y-axis at -2 : graph B

$y = -x^3$ is a cubic going down from left to right: graph A

$y = \left(\frac{1}{2}\right)^x$ is an exponential that gets smaller as x increases, through $(0, 1)$: graph D

$y = -\frac{1}{x}$ is a reciprocal graph (two branches), in the top-left and bottom-right: graph C

Answer: $y = x^2 - 2$: B; $y = -x^3$: A; $y = \left(\frac{1}{2}\right)^x$: D; $y = -\frac{1}{x}$: C

Tip: Check a point to decide between similar graphs. For $y = -x^3$, when $x = 1$, $y = -1$, so the graph is below the x -axis on the right.

Marks: 3 marks for all four correct. 2 marks for two or three correct. 1 mark for one correct.

2 (2 marks)

The graph of $y = (x + a)^2 + b$ has a turning point at $(4, -6)$
Write down the value of a and the value of b .

$(x + a)^2$ is never negative. It is smallest (0) when $x = -a$.

So the turning point is at $(-a, b)$.

$-a = 4$, so $a = -4$

$b = -6$

Check: $y = (x - 4)^2 - 6$ has its lowest point at $(4, -6)$ ✓

Answer: $a = -4$, $b = -6$

Tip: The x -coordinate of the turning point has the opposite sign to the number in the bracket:
 $(x - 4)^2$ gives $x = 4$.

Marks: 1 mark for $a = -4$. 1 mark for $b = -6$.

3 (2 marks)

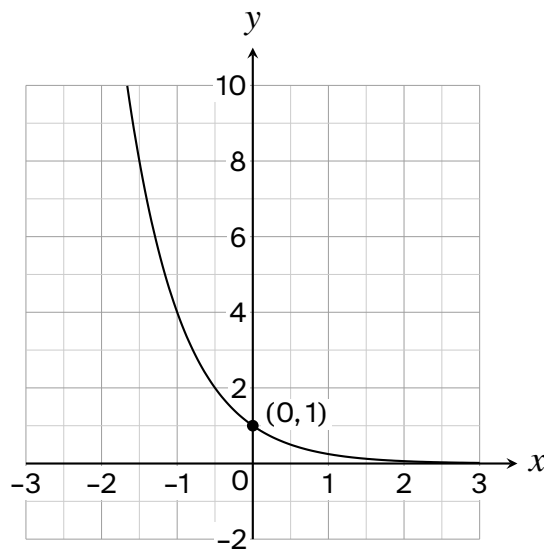
On the grid, draw a sketch of the graph of $y = 0.25^x$. Label the coordinates of the point where the graph meets the y -axis.

When $x = 0$: $0.25^0 = 1$, so the graph meets the y -axis at $(0, 1)$.

Some other points: $(-1, 4)$, $(-1.5, 8)$, $(1, 0.25)$, $(2, 0.0625)$

0.25 is less than 1, so y gets smaller as x increases.

For positive x the curve gets closer and closer to the x -axis but never meets it.



Answer: Decreasing exponential curve through (0,1), above the x -axis

Tip: When the number being powered is between 0 and 1, the graph goes **down** from left to right. It still passes through (0,1) and never touches the x -axis.

Marks: 1 mark for the correct shape (decreasing, above the x -axis). 1 mark for (0,1) labelled.

4 (3 marks)

The graph of $y = a \times b^x$ passes through the points (0,3) and (2,12)
Find the values of a and b , where $b > 0$

Substitute (0,3): $3 = a \times b^0$

$b^0 = 1$, so $a = 3$

Substitute (2,12): $12 = 3 \times b^2$

Divide by 3: $b^2 = 4$

$b > 0$, so $b = 2$

Answer: $a = 3$, $b = 2$

Marks: 1 mark for $a = 3$. 1 mark for $3b^2 = 12$. 1 mark for $b = 2$.

5 (2 marks)

The curve $y = k^x$, where $k > 1$, passes through $A(1.5, 8)$.

- (a) Find the value of k .

$A(1.5, 8)$ is on the curve: $k^{1.5} = 8$

$1.5 = \frac{3}{2}$, so $k^{\frac{3}{2}} = 8$

Raise both sides to the power $\frac{2}{3}$: $k = 8^{\frac{2}{3}}$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

Answer: $k = 4$

Tip: Check your value: $4^{1.5} = (\sqrt{4})^3 = 2^3 = 8$ ✓

- (b) The point B on the curve has x -coordinate -0.5 .
Find the y -coordinate of B .

$y = 4^{-0.5}$

A negative power means “one over”: $4^{-0.5} = \frac{1}{4^{0.5}}$

$4^{0.5} = \sqrt{4} = 2$

So $y = \frac{1}{2}$

Answer: $\frac{1}{2}$ (or 0.5)

Tip: $4^{-0.5}$ is not negative. A negative power gives a reciprocal, not a negative number.

6 (4 marks)

Sketch the graph of $y = x^2 - 6x + 5$

Show the coordinates of the turning point and any points where the graph crosses the axes.

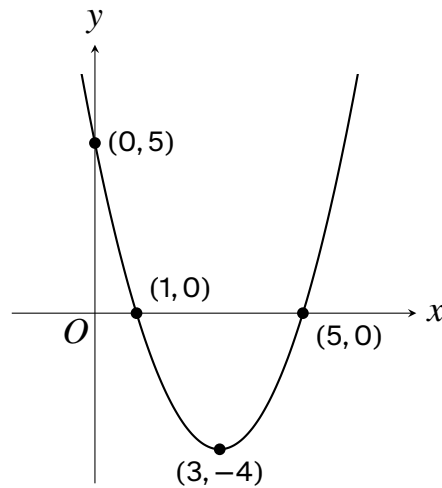
Factorise to find the roots: $x^2 - 6x + 5 = (x - 1)(x - 5)$

So it crosses the x -axis at $(1, 0)$ and $(5, 0)$.

When $x = 0$: $y = 5$, so the y -intercept is $(0, 5)$.

Complete the square for the turning point: $x^2 - 6x + 5 = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$

So the turning point is $(3, -4)$ (a minimum, as the graph is a U-shape).

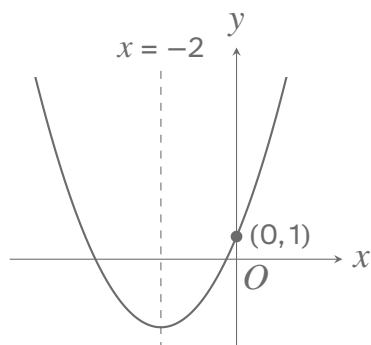


Answer: U-shape through $(1, 0)$, $(5, 0)$ and $(0, 5)$, with minimum point $(3, -4)$

Marks: 1 mark for a U-shape. 1 mark for the roots. 1 mark for the y -intercept. 1 mark for the turning point.

7 (3 marks)

Here is a sketch of a curve.



The curve has equation $y = x^2 + ax + b$

The line $x = -2$ is a line of symmetry of the curve.

The curve crosses the y -axis at $(0, 1)$.

Find the coordinates of the turning point of the curve.

The curve crosses the y -axis at $(0, 1)$, so when $x = 0$, $y = b = 1$.

The turning point lies on the line of symmetry, so its x -coordinate is -2 .

For $y = x^2 + ax + b$ the line of symmetry is $x = -\frac{a}{2}$, so $-\frac{a}{2} = -2$ and $a = 4$.

The curve is $y = x^2 + 4x + 1$

Substitute $x = -2$: $y = (-2)^2 + 4 \times (-2) + 1 = 4 - 8 + 1 = -3$

(Or complete the square: $x^2 + 4x + 1 = (x + 2)^2 - 3$)

Answer: $(-2, -3)$

Tip: The turning point is on the line of symmetry, so you already know its x -coordinate. You only need to work out y .

Marks: 1 mark for $b = 1$ or $a = 4$. 1 mark for substituting $x = -2$ into the correct equation. 1 mark for $(-2, -3)$.

8 (4 marks)

(a) Write $x^2 + 4x - 1$ in the form $(x + p)^2 + q$

Half of 4 is 2, so start with $(x + 2)^2$.

$(x + 2)^2 = x^2 + 4x + 4$, which is 5 too big.

So $x^2 + 4x - 1 = (x + 2)^2 - 4 - 1$

$= (x + 2)^2 - 5$

Answer: $(x + 2)^2 - 5$

Tip: Subtract the square of half the x -coefficient: $x^2 + 4x - 1 = (x + 2)^2 - 2^2 - 1$.

Marks: 1 mark for $(x + 2)^2$. 1 mark for $(x + 2)^2 - 5$.

- (b) Hence sketch the graph of $y = x^2 + 4x - 1$. Mark the turning point and the points where the graph meets the x -axis, giving their exact coordinates.

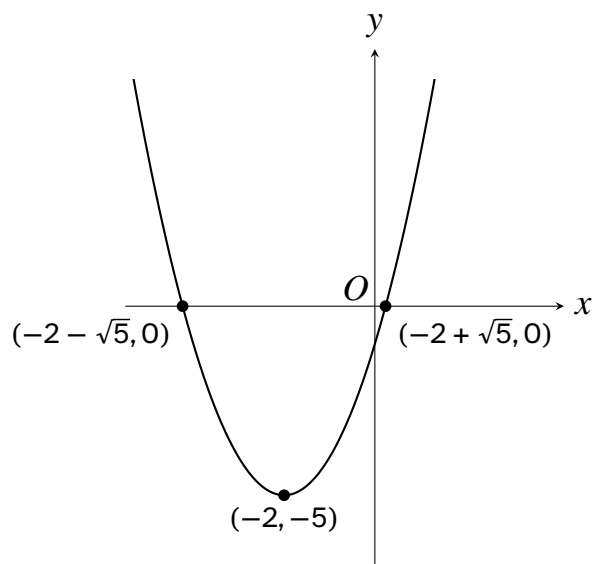
From part (a), $y = (x + 2)^2 - 5$, so the turning point is $(-2, -5)$.

The graph meets the x -axis when $(x + 2)^2 - 5 = 0$:

$(x + 2)^2 = 5$, so $x + 2 = \pm\sqrt{5}$

$x = -2 + \sqrt{5}$ or $x = -2 - \sqrt{5}$

(It also crosses the y -axis at $(0, -1)$.)



Answer: U-shape with minimum $(-2, -5)$, meeting the x -axis at $(-2 - \sqrt{5}, 0)$ and $(-2 + \sqrt{5}, 0)$

Tip: “Exact” means leave the answers as surds, $-2 \pm \sqrt{5}$. Decimals such as 0.236 are not exact.

Marks: 1 mark for a U-shape with turning point $(-2, -5)$. 1 mark for the x -intercepts $-2 \pm \sqrt{5}$.

9 (3 marks)

Sketch the graph of $y = (x - k)(x - 3k)$, where k is a positive constant. Show the coordinates of the turning point and of the points where the graph meets the axes, in terms of k .

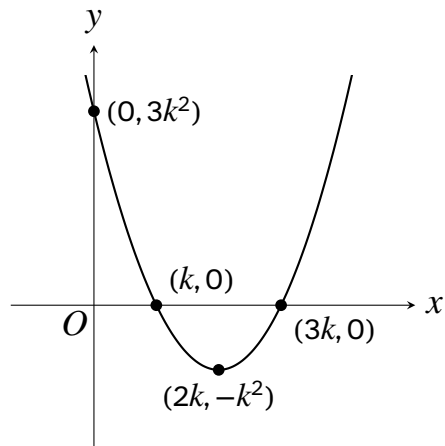
x -axis: when $y = 0$, $x = k$ or $x = 3k$. Both are positive, because $k > 0$.

y -axis: when $x = 0$, $y = (-k)(-3k) = 3k^2$, so $(0, 3k^2)$.

The turning point is halfway between the roots: $x = \frac{k + 3k}{2} = 2k$

$y = (2k - k)(2k - 3k) = k \times (-k) = -k^2$, so the turning point is $(2k, -k^2)$.

The x^2 term is positive, so the graph is a U-shape.



Answer: U-shape through $(k, 0)$, $(3k, 0)$ and $(0, 3k^2)$, with turning point $(2k, -k^2)$

Tip: $(-k) \times (-3k) = +3k^2$. Two negatives multiply to give a positive, so the y-intercept is above the origin.

Marks: 1 mark for a U-shape through $(k, 0)$ and $(3k, 0)$. 1 mark for $(0, 3k^2)$. 1 mark for the turning point $(2k, -k^2)$.
