

Rationalising Two-Term Denominators

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Rationalise the denominator and simplify fully

(a) $\frac{1}{\sqrt{3} + 1}$

Multiply the top and the bottom by $\sqrt{3} - 1$ (the same terms with the sign changed).

Bottom: $(\sqrt{3} + 1)(\sqrt{3} - 1) = 3 - \sqrt{3} + \sqrt{3} - 1 = 2$

Top: $1 \times (\sqrt{3} - 1) = \sqrt{3} - 1$

So the answer is $\frac{\sqrt{3} - 1}{2}$

Answer: $\frac{\sqrt{3} - 1}{2}$

Tip: Multiply by the denominator with its sign changed. Multiplying by $\sqrt{3}$ or by $\sqrt{3} + 1$ does not remove the surd from the bottom.

Marks: 1 mark for multiplying the top and bottom by $\sqrt{3} - 1$. 1 mark for $\frac{\sqrt{3}-1}{2}$.

(b) $\frac{4}{3 - \sqrt{5}}$

Multiply the top and the bottom by $3 + \sqrt{5}$.

Bottom: $(3 - \sqrt{5})(3 + \sqrt{5}) = 9 - 5 = 4$ (the $\sqrt{5}$ terms cancel)

Top: $4(3 + \sqrt{5})$

$$\frac{4(3 + \sqrt{5})}{4} = 3 + \sqrt{5}$$

Answer: $3 + \sqrt{5}$

Marks: 1 mark for multiplying the top and bottom by $3 + \sqrt{5}$. 1 mark for $3 + \sqrt{5}$.

2 (3 marks)

Show that $\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$ can be written as $3 + 2\sqrt{2}$

Multiply the top and the bottom by $\sqrt{2} + 1$.

Bottom: $(\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1$

Top: $(\sqrt{2} + 1)(\sqrt{2} + 1) = 2 + \sqrt{2} + \sqrt{2} + 1 = 3 + 2\sqrt{2}$

So the fraction is $\frac{3 + 2\sqrt{2}}{1} = 3 + 2\sqrt{2}$ ✓

Answer: $\frac{3 + 2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

Marks: 1 mark for multiplying the top and bottom by $\sqrt{2} + 1$. 1 mark for expanding the top to $3 + 2\sqrt{2}$. 1 mark for a denominator of 1 and completing the proof.

3 (3 marks)

Write $\frac{5 + \sqrt{3}}{2 - \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

Multiply the top and the bottom by $2 + \sqrt{3}$.

Bottom: $(2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$

Top: $(5 + \sqrt{3})(2 + \sqrt{3})$

$= 10 + 5\sqrt{3} + 2\sqrt{3} + 3$

$= 13 + 7\sqrt{3}$

Dividing by 1 changes nothing, so the answer is $13 + 7\sqrt{3}$.

Answer: $13 + 7\sqrt{3}$

Tip: Take care with $\sqrt{3} \times \sqrt{3}$: it is 3, not 9.

Marks: 1 mark for multiplying the top and bottom by $2 + \sqrt{3}$. 1 mark for a denominator of 1 or a correct expansion of the top. 1 mark for $13 + 7\sqrt{3}$.

4 (3 marks)

Show that $\frac{4 + \sqrt{18}}{\sqrt{2} + 1}$ can be written in the form $a + b\sqrt{2}$ where a and b are integers.

Simplify the surd first: $\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

So the fraction is $\frac{4 + 3\sqrt{2}}{\sqrt{2} + 1}$

Multiply the top and the bottom by $\sqrt{2} - 1$.

Bottom: $(\sqrt{2} + 1)(\sqrt{2} - 1) = 2 - 1 = 1$

Top: $(4 + 3\sqrt{2})(\sqrt{2} - 1) = 4\sqrt{2} - 4 + 6 - 3\sqrt{2}$
 $= 2 + \sqrt{2}$

So the fraction equals $2 + \sqrt{2}$, which is in the form $a + b\sqrt{2}$ ✓

Answer: $2 + \sqrt{2}$

Tip: Simplify any surds such as $\sqrt{18}$ first. It makes the expanding much easier.

Marks: 1 mark for multiplying the top and bottom by $\sqrt{2} - 1$. 1 mark for $4\sqrt{2} - 4 + 6 - 3\sqrt{2}$ (or equivalent). 1 mark for $2 + \sqrt{2}$.

5 (4 marks)

Show that $\frac{\sqrt{24} + 2}{4 + \sqrt{6}}$ can be written as $\frac{3\sqrt{6} - 2}{5}$

Simplify the surd first: $\sqrt{24} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$

So the fraction is $\frac{2\sqrt{6} + 2}{4 + \sqrt{6}}$

Multiply the top and the bottom by $4 - \sqrt{6}$.

$$\text{Bottom: } (4 + \sqrt{6})(4 - \sqrt{6}) = 16 - 6 = 10$$

$$\begin{aligned}\text{Top: } (2\sqrt{6} + 2)(4 - \sqrt{6}) &= 8\sqrt{6} - 12 + 8 - 2\sqrt{6} \\ &= 6\sqrt{6} - 4\end{aligned}$$

So the fraction is $\frac{6\sqrt{6} - 4}{10}$

Divide the top and the bottom by 2: $\frac{3\sqrt{6} - 2}{5}$ ✓

Answer: $\frac{3\sqrt{6} - 2}{5}$

Tip: Remember $2\sqrt{6} \times \sqrt{6} = 2 \times 6 = 12$. At the end, check whether the top and bottom share a factor. $\frac{6\sqrt{6}-4}{10}$ is not finished.

Marks: 1 mark for $2\sqrt{6}$ or multiplying by $\frac{4-\sqrt{6}}{4-\sqrt{6}}$. 1 mark for a denominator of 10. 1 mark for a numerator of $6\sqrt{6} - 4$. 1 mark for $\frac{3\sqrt{6}-2}{5}$.

6 (3 marks)

The area of a rectangle is 22 cm^2 .

The length of the rectangle is $(4 + \sqrt{5}) \text{ cm}$.

Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{5}$

$$\text{Width} = \text{area} \div \text{length} = \frac{22}{4 + \sqrt{5}}$$

Multiply the top and the bottom by $4 - \sqrt{5}$.

$$\text{Bottom: } (4 + \sqrt{5})(4 - \sqrt{5}) = 16 - 5 = 11$$

So the width is $\frac{22(4 - \sqrt{5})}{11}$

$22 \div 11 = 2$, so this is $2(4 - \sqrt{5}) = 8 - 2\sqrt{5}$

Answer: $8 - 2\sqrt{5}$ cm

Marks: 1 mark for $\frac{22}{4+\sqrt{5}}$. 1 mark for multiplying the top and bottom by $4 - \sqrt{5}$. 1 mark for $8 - 2\sqrt{5}$.

7 (3 marks)

Show that $\frac{1}{\sqrt{5} - 2} - \frac{1}{\sqrt{5} + 2} = 4$

Use a common denominator of $(\sqrt{5} - 2)(\sqrt{5} + 2)$, just like with ordinary fractions.

Common denominator: $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$

Top: $(\sqrt{5} + 2) - (\sqrt{5} - 2)$

$= \sqrt{5} + 2 - \sqrt{5} + 2 = 4$

So the expression is $\frac{4}{1} = 4$ ✓

Answer: $\frac{4}{1} = 4$

Marks: 1 mark for using a common denominator. 1 mark for $5 - 4$ (a denominator of 1). 1 mark for completing to 4.
