

Rationalising the Denominator

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (6 marks)

Rationalise the denominator of each fraction.
Simplify your answers.

(a) $\frac{5}{\sqrt{2}}$

Multiply the top and the bottom by $\sqrt{2}$:

$$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$\sqrt{2} \times \sqrt{2} = 2, \text{ so this is } \frac{5\sqrt{2}}{2}$$

Answer: $\frac{5\sqrt{2}}{2}$

Tip: Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$ (which equals 1), so you change the top **and** the bottom. Multiplying only the bottom changes the value.

Marks: 1 mark for $\frac{5\sqrt{2}}{\sqrt{2}\sqrt{2}}$. 1 mark for $\frac{5\sqrt{2}}{2}$.

(b) $\frac{14}{\sqrt{7}}$

Multiply the top and the bottom by $\sqrt{7}$:

$$\frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{7}}{7}$$

Simplify: $14 \div 7 = 2$, so the answer is $2\sqrt{7}$

Answer: $2\sqrt{7}$

Tip: Always cancel at the end. Leaving $\frac{14\sqrt{7}}{7}$ is not simplified and loses a mark.

Marks: 1 mark for $\frac{14\sqrt{7}}{7}$. 1 mark for $2\sqrt{7}$.

(c) $\frac{3}{2\sqrt{6}}$

Multiply the top and the bottom by $\sqrt{6}$:

$$\frac{3\sqrt{6}}{2 \times 6} = \frac{3\sqrt{6}}{12}$$

Simplify (divide top and bottom by 3): $\frac{\sqrt{6}}{4}$

Answer: $\frac{\sqrt{6}}{4}$

Marks: 1 mark for $\frac{3\sqrt{6}}{12}$. 1 mark for $\frac{\sqrt{6}}{4}$.

2 (2 marks)

Show that $\frac{40}{\sqrt{8}} = 10\sqrt{2}$

Simplify the surd first: $\sqrt{8} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$$\text{So } \frac{40}{\sqrt{8}} = \frac{40}{2\sqrt{2}} = \frac{20}{\sqrt{2}}$$

Multiply the top and the bottom by $\sqrt{2}$: $\frac{20\sqrt{2}}{2}$

$20 \div 2 = 10$, so this is $10\sqrt{2}$ ✓

Answer: $\frac{40}{2\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2}$

Tip: You can also multiply by $\frac{\sqrt{8}}{\sqrt{8}}$ to get $\frac{40\sqrt{8}}{8} = 5\sqrt{8}$, but then you must still simplify $\sqrt{8}$ to $2\sqrt{2}$.

Marks: 1 mark for $\sqrt{8} = 2\sqrt{2}$ or for multiplying the top and bottom by $\sqrt{8}$. 1 mark for a complete method leading to $10\sqrt{2}$.

3 (2 marks)

$$\frac{\sqrt{3} \times \sqrt{15}}{\sqrt{5}}$$

Simplify fully.

Top: $\sqrt{3} \times \sqrt{15} = \sqrt{45}$

Divide: $\frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9}$

$$\sqrt{9} = 3$$

Answer: 3

Marks: 1 mark for $\sqrt{45}$ or $\sqrt{9}$ (or $\sqrt{3} \times \sqrt{3}$). 1 mark for 3.

4 (2 marks)

The area of a rectangle is 12 cm^2 .

The length of the rectangle is $\sqrt{6} \text{ cm}$.

Find the width of the rectangle.

Give your answer in the form $a\sqrt{b}$

$$\text{Width} = \text{area} \div \text{length} = \frac{12}{\sqrt{6}}$$

$$\text{Rationalise: } \frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{6}$$

$12 \div 6 = 2$, so the width is $2\sqrt{6}$ cm

Answer: $2\sqrt{6}$ cm

Marks: 1 mark for $\frac{12}{\sqrt{6}}$ rationalised. 1 mark for $2\sqrt{6}$.

5 (3 marks)

Show that $3\sqrt{20} - \frac{15}{\sqrt{5}}$ can be written as $3\sqrt{5}$

Simplify the surd: $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$, so $3\sqrt{20} = 6\sqrt{5}$

Rationalise the fraction: $\frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5} = 3\sqrt{5}$

Subtract: $6\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$ ✓

Answer: $6\sqrt{5} - 3\sqrt{5} = 3\sqrt{5}$

Tip: $3\sqrt{20}$ means $3 \times \sqrt{20} = 3 \times 2\sqrt{5} = 6\sqrt{5}$. Do not forget to multiply by the 3.

Marks: 1 mark for $6\sqrt{5}$ (or $2\sqrt{5}$). 1 mark for $3\sqrt{5}$ from the fraction. 1 mark for completing to $3\sqrt{5}$.

6 (3 marks)

Express $\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{20}}$ in the form $\frac{a\sqrt{5}}{b}$ where a and b are integers.

First fraction: $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

Second fraction: $\sqrt{20} = 2\sqrt{5}$, so $\frac{1}{\sqrt{20}} = \frac{1}{2\sqrt{5}}$

$\frac{1}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{10}$

Use a common denominator of 10: $\frac{\sqrt{5}}{5} = \frac{2\sqrt{5}}{10}$

Add: $\frac{2\sqrt{5}}{10} + \frac{\sqrt{5}}{10} = \frac{3\sqrt{5}}{10}$

Answer: $\frac{3\sqrt{5}}{10}$

Marks: 1 mark for $\frac{\sqrt{5}}{5}$ or $2\sqrt{5}$ seen. 1 mark for $\frac{\sqrt{5}}{10}$ or a correct common denominator. 1 mark for $\frac{3\sqrt{5}}{10}$.

7 (3 marks)

Express $\frac{(\sqrt{2})^5}{\sqrt{6}}$ in the form $\frac{a\sqrt{b}}{c}$ where a and c are integers and b is a prime number.

$$(\sqrt{2})^5 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$$

Each pair $\sqrt{2} \times \sqrt{2} = 2$, so $(\sqrt{2})^5 = 2 \times 2 \times \sqrt{2} = 4\sqrt{2}$

So the expression is $\frac{4\sqrt{2}}{\sqrt{6}}$

$$\sqrt{6} = \sqrt{2} \times \sqrt{3}, \text{ so cancel } \sqrt{2}: \frac{4\sqrt{2}}{\sqrt{2} \times \sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\text{Rationalise: } \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$(\text{Or multiply the top and bottom by } \sqrt{6}: \frac{4\sqrt{2} \times \sqrt{6}}{6} = \frac{4\sqrt{12}}{6} = \frac{8\sqrt{3}}{6} = \frac{4\sqrt{3}}{3})$$

Answer: $\frac{4\sqrt{3}}{3}$

Tip: Two $\sqrt{2}$ s make 2, so $(\sqrt{2})^4 = 4$ and $(\sqrt{2})^5 = 4\sqrt{2}$. It is not 10 or $5\sqrt{2}$.

Marks: 1 mark for $(\sqrt{2})^5 = 4\sqrt{2}$. 1 mark for $\frac{4}{\sqrt{3}}$ or for multiplying the top and bottom by $\sqrt{6}$. 1 mark for $\frac{4\sqrt{3}}{3}$.
