

Ratio and Algebra

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

$$a:b = 2:3 \text{ and } a:c = 4:7$$

Find $b:c$

a is in both ratios, so make the a numbers the same.

$$a:b = 2:3 = 4:6 \text{ (multiply by 2)}$$

$$a:c = 4:7$$

$$\text{So } a:b:c = 4:6:7$$

$$b:c = 6:7$$

Answer: 6 : 7

Tip: Scale one ratio so that a is the same number in both. Do not just write 3:7 from the numbers you can see.

Marks: 1 mark for $a:b = 4:6$. 1 mark for 6 : 7.

2 (3 marks)

$$(3x + 2):(x + 4) = 5:3$$

Find the value of x .

Write the ratios as fractions: $\frac{3x + 2}{x + 4} = \frac{5}{3}$

$$\text{Cross-multiply: } 3(3x + 2) = 5(x + 4)$$

$$9x + 6 = 5x + 20$$

$$4x = 14$$

$$x = 3.5$$

Check: $3x + 2 = 12.5$ and $x + 4 = 7.5$, and $12.5:7.5 = 5:3$ ✓

Answer: 3.5

Marks: 1 mark for $3(3x + 2) = 5(x + 4)$. 1 mark for $4x = 14$. 1 mark for 3.5.

3 (3 marks)

$$\frac{x}{y} = \frac{3}{4}$$

Find the value of $\frac{2x + y}{3x - y}$

$$\frac{x}{y} = \frac{3}{4}, \text{ so let } x = 3k \text{ and } y = 4k.$$

$$2x + y = 6k + 4k = 10k$$

$$3x - y = 9k - 4k = 5k$$

$$\frac{10k}{5k} = 2$$

Answer: 2

Tip: You can simply use $x = 3$ and $y = 4$: $\frac{6+4}{9-4} = \frac{10}{5} = 2$. Any pair in the ratio 3:4 gives the same answer.

Marks: 1 mark for $x = 3k$ and $y = 4k$ (or using $x = 3$ and $y = 4$). 1 mark for $\frac{10k}{5k}$ or $\frac{10}{5}$. 1 mark for 2.

4 (5 marks)

(a) $\frac{c + 2d}{3} = \frac{2c - d}{2}$

Find the ratio $c:d$

$$\text{Cross-multiply: } 2(c + 2d) = 3(2c - d)$$

$$2c + 4d = 6c - 3d$$

$$\text{Collect the } c \text{ terms on one side and the } d \text{ terms on the other: } 7d = 4c$$

$$\frac{c}{d} = \frac{7}{4}$$

So $c:d = 7:4$

Answer: 7 : 4

Tip: $4c = 7d$ gives $c:d = 7:4$, not $4:7$ (the numbers swap sides). Check: $c = 7, d = 4$ gives $\frac{15}{3} = 5$ and $\frac{10}{2} = 5$ ✓

Marks: 1 mark for $4c = 7d$. 1 mark for $7 : 4$.

- (b) $(x + y)^2 = 3xy + 7y^2$ where x and y are both positive.
Find the ratio $x:y$

Expand the bracket: $x^2 + 2xy + y^2 = 3xy + 7y^2$

Move everything to one side: $x^2 - xy - 6y^2 = 0$

Factorise (just like $x^2 - x - 6$): $(x - 3y)(x + 2y) = 0$

So $x - 3y = 0$ or $x + 2y = 0$

x and y are both positive, so $x + 2y$ cannot be 0.

So $x = 3y$, which gives $x:y = 3:1$

Answer: 3 : 1

Tip: $(x + y)^2$ is $x^2 + 2xy + y^2$, not $x^2 + y^2$. Then factorise it like a quadratic (or divide every term by y^2 and solve for $\frac{x}{y}$).

Marks: 1 mark for expanding and rearranging to $x^2 - xy - 6y^2 = 0$. 1 mark for factorising correctly. 1 mark for $3 : 1$.

5 (4 marks)

A bag contains black and white beads in the ratio 4:5

Some black beads are removed. The ratio becomes 2:5

12 black beads were removed.

How many beads are now in the bag?

Let the number of black beads be $4k$ and white be $5k$.

12 black beads are removed: black = $4k - 12$. White stays $5k$.

$$\frac{4k - 12}{5k} = \frac{2}{5}$$

Cross-multiply: $5(4k - 12) = 2 \times 5k$

$$20k - 60 = 10k$$

$$10k = 60$$

$$k = 6$$

Now: black = $4 \times 6 - 12 = 12$ and white = $5 \times 6 = 30$

Beads now = $12 + 30 = 42$

Answer: 42

Marks: 1 mark for $4k - 12$. 1 mark for a correct equation. 1 mark for $k = 6$. 1 mark for 42.

6 (3 marks)

The ratio $(x + 2y) : (x - y)$ is equivalent to $3 : k$

Show that $x = \frac{y(2k + 3)}{3 - k}$

Write the ratio as an equation: $\frac{x + 2y}{x - y} = \frac{3}{k}$

Cross-multiply: $k(x + 2y) = 3(x - y)$

$$kx + 2ky = 3x - 3y$$

Get the x terms on one side and the y terms on the other:

$$2ky + 3y = 3x - kx$$

Factorise both sides: $y(2k + 3) = x(3 - k)$

Divide both sides by $(3 - k)$: $x = \frac{y(2k + 3)}{3 - k}$

Answer: Shown

Tip: Keep the ratio the right way round: $(x + 2y)$ goes with 3 and $(x - y)$ goes with k . Take care with signs when you move terms across.

Marks: 1 mark for $k(x + 2y) = 3(x - y)$. 1 mark for collecting the x terms on one side. 1 mark for a fully correct method.

7 (5 marks)

Ines and Mateo share some money in the ratio 5:3

Ines then gives £40 of her money to Mateo.

Mateo then spends £20

The amounts of money Ines and Mateo now have are in the ratio 3:4

How much money did Ines have at the start?

At the start: Ines has $5k$ and Mateo has $3k$.

Ines gives £40 to Mateo: Ines has $5k - 40$ and Mateo has $3k + 40$.

Mateo spends £20: Mateo has $3k + 40 - 20 = 3k + 20$.

$$\frac{5k - 40}{3k + 20} = \frac{3}{4}$$

Cross-multiply: $4(5k - 40) = 3(3k + 20)$

$$20k - 160 = 9k + 60$$

$$11k = 220$$

$$k = 20$$

Ines had $5 \times 20 = £100$ at the start.

Check: Ines £60 and Mateo £80, and $60:80 = 3:4$ ✓

Answer: £100

Tip: The £40 is taken from Ines **and** added to Mateo. Then take off the £20 Mateo spends before you write the new ratio.

Marks: 1 mark for $5k - 40$. 1 mark for $3k + 20$. 1 mark for a correct equation. 1 mark for $k = 20$. 1 mark for £100.

8 (4 marks)

x and y are both positive.

- (a) $(x^2 + 5y^2):(x^2 - y^2) = 7:5$
Find the ratio $x:y$

Write the ratio as an equation: $\frac{x^2 + 5y^2}{x^2 - y^2} = \frac{7}{5}$

Cross-multiply: $5(x^2 + 5y^2) = 7(x^2 - y^2)$

$$5x^2 + 25y^2 = 7x^2 - 7y^2$$

Collect like terms: $32y^2 = 2x^2$, so $x^2 = 16y^2$

Square root (both are positive): $x = 4y$

$$x:y = 4:1$$

Answer: 4 : 1

Tip: 16:1 is the ratio $x^2:y^2$. Remember to square root to get $x:y$.

Marks: 1 mark for multiplying out the brackets. 1 mark for $x^2 = 16y^2$ (or $2x^2 = 32y^2$). 1 mark for 4 : 1.

- (b) Given also that $x + y = 25$, find the value of x .

$x:y = 4:1$, so x is $\frac{4}{5}$ of $x + y$.

$$x = \frac{4}{5} \times 25 = 20$$

Answer: 20
