

Quadratic Inequalities

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

(a) Solve $x^2 < 49$

Solve $x^2 = 49$ first: $x = 7$ or $x = -7$

$x^2 < 49$ when x is between -7 and 7 .

Check: $x = 0$ gives $0 < 49$ ✓

Answer: $-7 < x < 7$

Tip: Remember the negative square root. $x < 7$ is wrong: for example, $x = -8$ gives 64 , which is not less than 49 .

Marks: 1 mark for ± 7 . 1 mark for $-7 < x < 7$.

(b) Solve $5x^2 > 45$

Divide both sides by 5 :

$$x^2 > 9$$

Solve $x^2 = 9$: $x = 3$ or $x = -3$

$x^2 > 9$ when x is more than 3 , or when x is less than -3 .

Answer: $x < -3$ or $x > 3$

Marks: 1 mark for ± 3 . 1 mark for $x < -3$ or $x > 3$.

2 (3 marks)

Solve $x^2 - 9x + 14 < 0$

Factorise: $x^2 - 9x + 14 = (x - 2)(x - 7)$

The critical values (where it equals 0) are $x = 2$ and $x = 7$.

Sketch $y = x^2 - 9x + 14$: a U-shaped curve crossing the x -axis at 2 and 7.

< 0 means below the x -axis.

This happens between 2 and 7.

Answer: $2 < x < 7$

Tip: Don't give $x = 2$ and $x = 7$ as your final answer. Sketch the curve, then write an inequality.

Marks: 1 mark for factorising. 1 mark for the critical values 2 and 7. 1 mark for $2 < x < 7$.

3 (3 marks)

Solve $3x^2 + 5x - 2 \geq 0$

Factorise: $3x^2 + 5x - 2 = (3x - 1)(x + 2)$

Critical values:

$$3x - 1 = 0 \text{ gives } x = \frac{1}{3}$$

$$x + 2 = 0 \text{ gives } x = -2$$

Sketch: a U-shaped curve crossing the x -axis at -2 and $\frac{1}{3}$.

≥ 0 means on or above the x -axis, which is outside the two roots.

Answer: $x \leq -2$ or $x \geq \frac{1}{3}$

Tip: Take care finding the critical values from the brackets: $(x + 2) = 0$ gives $x = -2$, not $x = 2$.

Marks: 1 mark for factorising. 1 mark for the critical values -2 and $\frac{1}{3}$. 1 mark for the final answer.

4 (3 marks)

Solve $2x^2 \leq 18$

Write your answer using set notation.

Divide both sides by 2:

$$x^2 \leq 9$$

Solve $x^2 = 9$: $x = \pm 3$

$x^2 \leq 9$ between -3 and 3 , including both ends.

$$-3 \leq x \leq 3$$

In set notation: $\{x: -3 \leq x \leq 3\}$

Answer: $\{x: -3 \leq x \leq 3\}$

Marks: 1 mark for ± 3 . 1 mark for $-3 \leq x \leq 3$. 1 mark for writing it in set notation.

5 (3 marks)

Solve $(3 - 2x)(x + 4) < 0$

It is already factorised, so find the critical values.

$$3 - 2x = 0 \text{ gives } x = \frac{3}{2}$$

$$x + 4 = 0 \text{ gives } x = -4$$

Multiplying out the brackets gives $-2x^2 + \dots$, so the x^2 term is negative.

So $y = (3 - 2x)(x + 4)$ is an $\cap$ -shaped curve crossing the x -axis at -4 and $\frac{3}{2}$.

< 0 means below the x -axis.

An $\cap$ -shaped curve is below the x -axis outside the two critical values.

Check with $x = 0$: $3 \times 4 = 12$, which is not less than 0, so $x = 0$ (between the values) is not a solution ✓

Answer: $x < -4$ or $x > \frac{3}{2}$

Tip: Look at the sign of the x^2 term. Here it is negative, so the curve is $\cap$ -shaped and the answer is **outside** the critical values, not between them. Test a value such as $x = 0$ to check.

Marks: 1 mark for the critical values -4 and $\frac{3}{2}$. 1 mark for a correct sketch or a method that identifies the correct regions. 1 mark for $x < -4$ or $x > \frac{3}{2}$.

6 (4 marks)

(a) Factorise $6x^2 + x - 12$

Multiply the first and last numbers: $6 \times (-12) = -72$

Find two numbers that multiply to -72 and add to 1: 9 and -8

Split the middle term:

$$6x^2 + 9x - 8x - 12$$

Factorise in pairs:

$$3x(2x + 3) - 4(2x + 3)$$

$$= (3x - 4)(2x + 3)$$

$$\text{Check: } (3x - 4)(2x + 3) = 6x^2 + 9x - 8x - 12 = 6x^2 + x - 12 \quad \checkmark$$

Answer: $(3x - 4)(2x + 3)$

Marks: 1 mark for a correct method, e.g. $3x(2x + 3) - 4(2x + 3)$. 1 mark for $(3x - 4)(2x + 3)$.

(b) Hence, or otherwise, solve $6x^2 + x - 12 < 0$

Use part (a): $(3x - 4)(2x + 3) < 0$

Critical values:

$$3x - 4 = 0 \text{ gives } x = \frac{4}{3}$$

$$2x + 3 = 0 \text{ gives } x = -\frac{3}{2}$$

The U-shaped curve is below the x -axis between these values.

Answer: $-\frac{3}{2} < x < \frac{4}{3}$

Tip: “Hence” means use your answer to part (a). You don’t need to start again.

Marks: 1 mark for the critical values $\frac{4}{3}$ and $-\frac{3}{2}$. 1 mark for $-\frac{3}{2} < x < \frac{4}{3}$.

7 (3 marks)

Solve $(x - 2)^2 < \frac{9}{16}$

Square root both sides. The root can be positive or negative:

$$-\frac{3}{4} < x - 2 < \frac{3}{4}$$

Add 2 to all three parts:

$$2 - \frac{3}{4} < x < 2 + \frac{3}{4}$$

$$\frac{5}{4} < x < \frac{11}{4}$$

(Or expand and rearrange: $16x^2 - 64x + 55 < 0$, so $(4x - 5)(4x - 11) < 0$.)

Answer: $\frac{5}{4} < x < \frac{11}{4}$

Tip: Many students only use $+\frac{3}{4}$. Remember the negative root as well, so there are two critical values.

Marks: 1 mark for $x - 2 = \pm\frac{3}{4}$ (or rearranging to a quadratic). 1 mark for $\frac{5}{4}$ and $\frac{11}{4}$. 1 mark for $\frac{5}{4} < x < \frac{11}{4}$.

8 (4 marks)

Find the set of values of x for which $x^2 - 4x - 12 > 0$ and $2x + 1 > 3$

First inequality. Factorise: $x^2 - 4x - 12 = (x - 6)(x + 2)$

The critical values are 6 and -2 .

> 0 means outside the roots: $x < -2$ or $x > 6$

Second inequality:

$$2x + 1 > 3$$

$$2x > 2$$

$$x > 1$$

Now find the values that fit both.

$x < -2$ does not fit with $x > 1$.

Every number above 6 is also above 1, so $x > 6$ fits both.

Answer: $x > 6$

Tip: Show both answers on one number line to see where they overlap. The final answer must satisfy both inequalities.

Marks: 1 mark for factorising. 1 mark for $x < -2$ or $x > 6$. 1 mark for $x > 1$. 1 mark for $x > 6$.

9 (3 marks)

Solve the inequality $2x(x - 1) \leq 3(x + 1)$

Expand both brackets:

$$2x^2 - 2x \leq 3x + 3$$

Rearrange so that one side is 0:

$$2x^2 - 5x - 3 \leq 0$$

Factorise: $(2x + 1)(x - 3) \leq 0$

The critical values are $x = -\frac{1}{2}$ and $x = 3$.

The U-shaped curve $y = 2x^2 - 5x - 3$ is on or below the x -axis between these values.

Answer: $-\frac{1}{2} \leq x \leq 3$

Tip: Expand and collect everything on one side before you factorise. $2x(x - 1)$ is $2x^2 - 2x$, not $2x^2 - 1$.

Marks: 1 mark for $2x^2 - 5x - 3 \leq 0$. 1 mark for the critical values $-\frac{1}{2}$ and 3. 1 mark for $-\frac{1}{2} \leq x \leq 3$.
