

Proving Circle Theorems

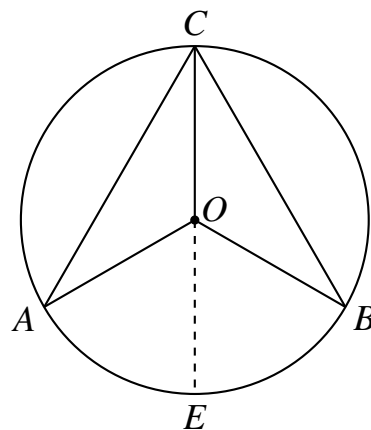
Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

A, B and C are points on the circumference of a circle, centre O.



*Diagram **NOT** accurately drawn*

Prove that angle $AOB = 2 \times$ angle ACB .

You may use the diagram, where CO is extended to E .

Let angle $OCA = a$ and angle $OCB = b$.

So angle $ACB = a + b$.

$OA = OC$ (radii), so triangle OAC is an isosceles triangle.

So angle $OAC = a$ (base angles of an isosceles triangle are equal).

Angle $AOE = a + a = 2a$

(The exterior angle of a triangle equals the sum of the two interior opposite angles.)

In the same way, $OB = OC$, so angle $OBC = b$ and angle $BOE = 2b$.

Angle $AOB = 2a + 2b = 2(a + b)$

So angle $AOB = 2 \times \text{angle } ACB$.

Tip: Use letters, not numbers. Showing it works for one set of angles does not prove it.

Marks: 1 mark for the isosceles triangles (radii). 1 mark for the exterior angle $2a$. 1 mark for angle $AOB = 2a + 2b$. 1 mark for a complete proof with reasons.

2 (4 marks)

P , Q and R are points on a circle, centre O .

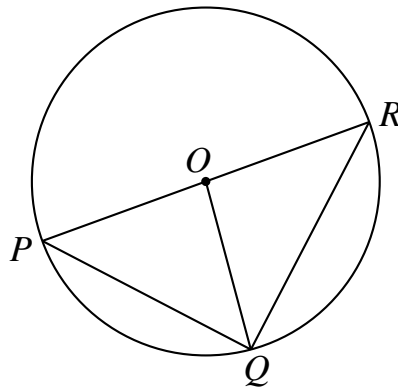


Diagram **NOT** accurately drawn

POR is a straight line.

Prove that triangle PQR has a right angle at Q .

Give a reason for each stage of your working.

POR is a straight line through the centre, so PR is a diameter.

$OP = OQ = OR$ (radii of the circle)

Triangle OPQ is isosceles, so angle $OPQ = \text{angle } OQP$. Call this p .

Triangle OQR is isosceles, so angle $ORQ = \text{angle } OQR$. Call this q .

The angles of triangle PQR are p , q and $p + q$.

Angles in a triangle add up to 180° : $p + q + (p + q) = 180$

$2(p + q) = 180$, so $p + q = 90$

Angle $PQR = p + q = 90^\circ$, so the triangle has a right angle at Q .

Tip: You are proving the theorem, so you cannot quote “the angle in a semicircle is 90° ”. Write “isosceles triangle” and say which sides are equal and why.

Marks: 1 mark for the radii being equal. 1 mark for the base angles of the isosceles triangles. 1 mark for $2p + 2q = 180$. 1 mark for a complete proof.

3 (3 marks)

A, B, C and D are points on a circle.

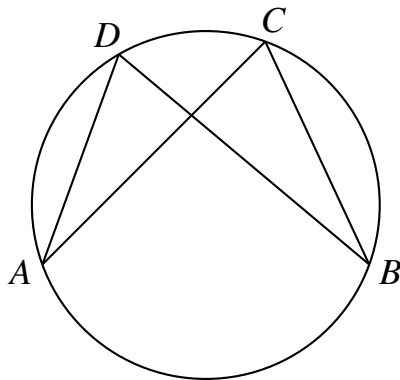


Diagram **NOT** accurately drawn

Prove that angle $ACB = \text{angle } ADB$.

You may use the fact that the angle at the centre is twice the angle at the circumference.

Let O be the centre of the circle.

Angle $AOB = 2 \times \text{angle } ACB$

Angle $AOB = 2 \times \text{angle } ADB$

(The angle at the centre is twice the angle at the circumference.)

Both are equal to half of angle AOB .

So angle $ACB = \text{angle } ADB$.

Marks: 1 mark for angle $AOB = 2 \times \text{angle } ACB$. 1 mark for angle $AOB = 2 \times \text{angle } ADB$. 1 mark for the conclusion.

4 (3 marks)

$ABCD$ is a cyclic quadrilateral. O is the centre of the circle.

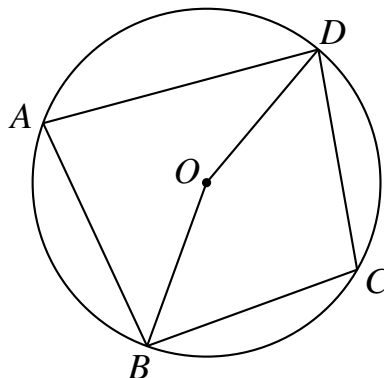


Diagram **NOT** accurately drawn

Prove that the opposite angles of a cyclic quadrilateral add up to 180°

You may use the fact that the angle at the centre is twice the angle at the circumference.

Let angle $BAD = x$ and angle $BCD = y$.

The angle at the centre on the same arc as x is $2x$.

The reflex angle at the centre on the same arc as y is $2y$.

(The angle at the centre is twice the angle at the circumference.)

These two angles at O make a full turn: $2x + 2y = 360$

(Angles around a point add up to 360° .)

Divide by 2: $x + y = 180$

Marks: 1 mark for the angles $2x$ and $2y$ at the centre. 1 mark for $2x + 2y = 360$ (angles at a point). 1 mark for $x + y = 180$.

5 (4 marks)

A and B are points on a circle, centre O .

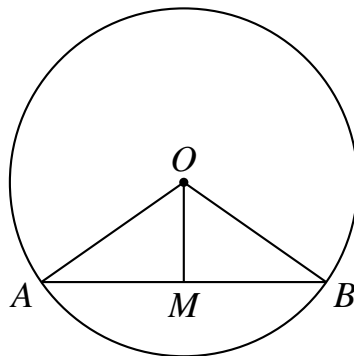


Diagram **NOT** accurately drawn

M is the midpoint of the chord AB .

Prove that OM is perpendicular to AB .

Look at triangles OAM and OBM .

$OA = OB$ (radii)

$AM = MB$ (M is the midpoint of AB)

OM is a side of both triangles (common side).

So triangles OAM and OBM are congruent (SSS).

So angle $OMA =$ angle OMB (corresponding angles of congruent triangles).

They lie on the straight line AB , so they add up to 180° .

So each angle is 90° , and OM is perpendicular to AB .

Tip: You need all three pairs of equal sides for SSS. One pair of equal sides is not enough.

Marks: 1 mark for pairs of equal sides with reasons. 1 mark for congruent (SSS). 1 mark for angle $OMA =$ angle OMB . 1 mark for showing each is 90° .

6 (3 marks)

A , B and C are points on a circle, centre O .

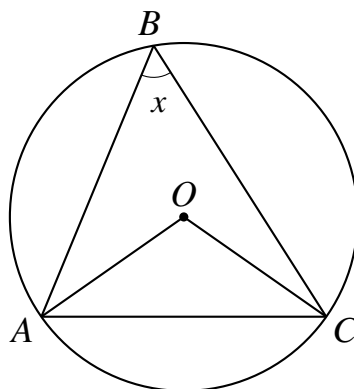


Diagram **NOT** accurately drawn

Angle $ABC = x$

Prove that angle $OCA = 90^\circ - x$

Give reasons for each stage of your working.

Angle $AOC = 2x$

(The angle at the centre is twice the angle at the circumference.)

$OA = OC$ (radii), so triangle OAC is an isosceles triangle.

So angle $OCA = \text{angle } OAC$ (base angles of an isosceles triangle are equal).

Angles in a triangle add up to 180° :

$$\text{Angle } OCA = \frac{1}{2}(180 - 2x) = 90 - x$$

Tip: Every step needs a reason with the key words, e.g. “the angle at the centre is twice the angle at the circumference”.

Marks: 1 mark for angle $AOC = 2x$ with a reason. 1 mark for the isosceles triangle with a reason. 1 mark for $90 - x$ with all reasons.

7 (4 marks)

TA and TB are tangents to a circle, centre O , from the point T .

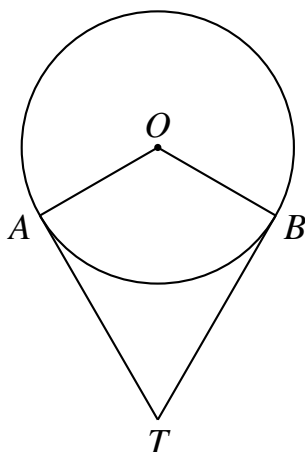


Diagram **NOT** accurately drawn

Prove that $TA = TB$.

Look at triangles OAT and OBT .

$OA = OB$ (radii)

OT is a side of both triangles (common side, the hypotenuse).

Angle $OAT = \text{angle } OBT = 90^\circ$ (a tangent is perpendicular to the radius).

So triangles OAT and OBT are congruent (RHS).

So $TA = TB$ (corresponding sides of congruent triangles).

Marks: 1 mark for $OA = OB$ (radii). 1 mark for the right angles with a reason. 1 mark for congruent (RHS). 1 mark for the conclusion $TA = TB$.