

Product Rule for Counting

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (1 mark)

A shop sells phone cases.

Each phone case is one of 9 colours and has one of 6 patterns.

How many different phone cases are possible?

Product rule: 9 choices of colour and 6 choices of pattern.

$$9 \times 6 = 54$$

Answer: 54

Tip: Multiply the numbers of choices, do not add them. $9 + 6 = 15$ is wrong.

2 (2 marks)

A car number plate has 2 letters followed by 3 digits.

The letters can be any of the 26 letters and the digits can be any from 0 to 9

How many different number plates are possible?

Each letter: 26 choices.

Each digit: 10 choices.

$$26 \times 26 \times 10 \times 10 \times 10 = 676\,000$$

Answer: 676 000

Marks: 1 mark for $26 \times 26 \times 10 \times 10 \times 10$. 1 mark for 676 000.

3 (2 marks)

Amir has 5 books. He puts 3 of them in a row on a shelf.
How many different ways can he do this?

First place on the shelf: 5 choices of book.

Second place: 4 choices (one book is already used).

Third place: 3 choices.

$$5 \times 4 \times 3 = 60$$

Answer: 60

Marks: 1 mark for $5 \times 4 \times 3$. 1 mark for 60.

4 (3 marks)

- (a) How many two-digit numbers can be made from the digits 1, 3, 5 and 7 if no digit is repeated?

First digit: 4 choices.

Second digit: 3 choices (no repeats).

$$4 \times 3 = 12$$

Answer: 12

Marks: 1 mark for 4×3 . 1 mark for 12.

- (b) How many of these numbers are greater than 50?

To be greater than 50, the first digit must be 5 or 7: 2 choices.

Second digit: 3 choices (any of the other three digits).

$$2 \times 3 = 6$$

(They are 51, 53, 57, 71, 73, 75.)

Answer: 6

5 (4 marks)

A computer game gives each player a username of 3 letters.
The letters are chosen from

A B C D E F G H

For example, CAB is one username.

- (a) Letters may be used more than once.
Work out the number of different usernames that can be made.

Each of the 3 letters can be any of the 8 letters.

$$8 \times 8 \times 8 = 512$$

Answer: 512

Marks: 1 mark for $8 \times 8 \times 8$ or 8^3 . 1 mark for 512.

- (b) Now each letter can be used only once.
The first letter of the username must be a vowel.
How many different usernames are possible?

First letter: a vowel, so A or E. That is 2 choices.

Second letter: 7 choices (any letter except the one already used).

Third letter: 6 choices.

$$2 \times 7 \times 6 = 84$$

Answer: 84

Tip: Deal with the letter that has a condition first. Then the number of choices goes down by 1 for each letter after it.

Marks: 1 mark for $2 \times 7 \times 6$. 1 mark for 84.

6 (2 marks)

Some people take part in a table-tennis tournament.
Each player plays every other player exactly once.
A total of 36 games are played.
Work out the number of players in the tournament.

With n players, each player plays $n - 1$ others: $n \times (n - 1)$

This counts every game twice (A v B and B v A are the same game), so the number of games is $\frac{n(n-1)}{2}$

$$\frac{n(n-1)}{2} = 36, \text{ so } n(n-1) = 72$$

$$9 \times 8 = 72, \text{ so } n = 9$$

Answer: 9

Tip: Remember to halve: $n \times (n-1)$ counts every game twice. Using $n(n-1) = 36$ gives no whole-number answer.

Marks: 1 mark for $n(n-1) = 72$ or a trial such as $\frac{9 \times 8}{2}$. 1 mark for 9.

7 (2 marks)

A drama group has 11 actors.

The director chooses one actor to play the hero and a different actor to play the villain.

Ifeoma says,

“There are 121 different ways to choose the hero and the villain.”

Is Ifeoma correct?

You must give a reason for your answer.

Hero: 11 choices of actor.

Villain: 10 choices (it must be a different actor).

$$11 \times 10 = 110 \text{ ways}$$

$$11 \times 11 = 121 \text{ would let the same actor play both parts.}$$

So Ifeoma is not correct. There are 110 ways.

Answer: No, there are $11 \times 10 = 110$ ways

Tip: Once one actor has been chosen for the first part, there is one fewer actor left for the second part.

Marks: 1 mark for 11×10 . 1 mark for No with 110.

8 (2 marks)

An ice-cream van sells ice cream in a cone or in a tub.

There are 6 flavours and 4 sauces.

A customer chooses one flavour and either a cone or a tub.

They then choose one sauce or they choose no sauce.

Show that there are 60 different ways to choose an ice cream.

Flavour: 6 choices.

Cone or tub: 2 choices.

Sauce: 4 sauces **or** no sauce, so $4 + 1 = 5$ choices.

$$6 \times 2 \times 5 = 60 \quad \checkmark$$

Answer: $6 \times 2 \times 5 = 60$

Tip: “No sauce” is one more choice, so there are 5 sauce choices, not 4. $6 \times 2 \times 4 = 48$ misses the ice creams with no sauce.

Marks: 1 mark for 5 sauce choices or $6 \times 2 = 12$. 1 mark for 60 from $6 \times 2 \times 5$.

9 (2 marks)

A password is made of 3 letters from A to Z followed by 2 digits from 0 to 9

Letters cannot be repeated. Digits can be repeated.

Show that there are more than 1.5 million possible passwords.

Letters (no repeats): 26, then 25, then 24 choices.

Digits (repeats allowed): 10 and 10 choices.

$$26 \times 25 \times 24 \times 10 \times 10 = 1\,560\,000$$

1 560 000 is more than 1 500 000 (1.5 million) $\checkmark$

Answer: $1\,560\,000 > 1\,500\,000$

Marks: 1 mark for $26 \times 25 \times 24 \times 10 \times 10$. 1 mark for 1 560 000 with a comparison to 1.5 million.

10 (3 marks)

A cycling club has 18 senior members and 7 junior members.

One senior member is chosen to be captain.

A different senior member is chosen to be vice-captain.

One junior member is chosen to be junior representative.

Work out the number of different ways these three people can be chosen.

Captain: 18 choices of senior member.

Vice-captain: 17 choices (a different senior member).

$$18 \times 17 = 306$$

Junior representative: 7 choices.

All three people are chosen, so multiply: $306 \times 7 = 2142$

Answer: 2142

Tip: The captain **and** the vice-captain **and** the junior representative are all chosen, so multiply.
Do not add 18×17 and 7.

Marks: 1 mark for 18×17 . 1 mark for $18 \times 17 \times 7$. 1 mark for 2142.
