

Probability - Sample Spaces and Two-Way Tables - Higher

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Ivan rolls two fair six-sided dice.

(a) Work out the probability that the total of the two scores is 9

There are $6 \times 6 = 36$ equally likely outcomes.

Total 9: (3,6), (4,5), (5,4), (6,3)

That is 4 of the 36 outcomes.

Answer: $\frac{4}{36}$

(b) Work out the probability that at least one of the dice lands on 6

A 6 on the first dice: 6 outcomes.

A 6 on the second dice: 6 outcomes.

(6,6) is in both lists, so only count it once: $6 + 6 - 1 = 11$

(Or: $1 - P(\text{no } 6) = 1 - \frac{25}{36} = \frac{11}{36}$.)

Answer: $\frac{11}{36}$

2 (4 marks)

The two-way table shows information about the members of a club.

	Swimming	Tennis	Total
Male	18		40
Female		12	
Total			70

A member who plays tennis is chosen at random.
Work out the probability that this member is female.

Complete the table.

Male tennis: $40 - 18 = 22$

Female total: $70 - 40 = 30$

Female swimming: $30 - 12 = 18$

Tennis total: $22 + 12 = 34$

The member is chosen from the 34 tennis players.

12 of them are female.

Answer: $\frac{12}{34}$

Tip: The member is chosen from the tennis players only, so the bottom of the fraction is 34, not 70.

Marks: 1 mark for 22 male tennis players. 1 mark for the tennis total of 34. 1 mark for a fraction out of 34. 1 mark for $\frac{12}{34}$ or an equivalent.

3 (3 marks)

Here are five number cards.

2	4	5	7	8
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Mateo takes three of the cards at random, one at a time.

He places them in a row, in the order he takes them, to make a 3-digit number.

- (a) How many different 3-digit numbers can Mateo make?

First card: 5 choices. Second card: 4 choices. Third card: 3 choices.

$$5 \times 4 \times 3 = 60$$

Answer: 60

- (b) Work out the probability that Mateo's number is odd **and** greater than 700

Greater than 700: the first digit must be 7 or 8

Odd: the last digit must be 5 or 7

First digit 7: the last digit must be 5, and the middle digit is 2, 4 or 8, so 3 numbers.

First digit 8: the last digit is 5 or 7, and the middle digit is one of the 3 cards left, so $2 \times 3 = 6$ numbers.

That is $3 + 6 = 9$ of the 60 equally likely numbers.

Answer: $\frac{9}{60}$ $\left(= \frac{3}{20} \right)$

Tip: A card cannot be used twice. If the first card is 7, the last card can only be 5.

Marks: 1 mark for counting the numbers starting with 7 or with 8 correctly (3 or 6). 1 mark for $\frac{9}{60}$ or an equivalent.

4 (2 marks)

A florist makes posies.

Each posy contains exactly two different types of flower.

The florist wants to be able to make at least 30 different posies.

Work out the least number of types of flower she needs.

With n types of flower, the number of different pairs is $\frac{n(n-1)}{2}$ (divide by 2 because rose with tulip is the same posy as tulip with rose).

$$n = 8: \frac{8 \times 7}{2} = 28, \text{ which is less than } 30$$

$$n = 9: \frac{9 \times 8}{2} = 36, \text{ which is at least } 30$$

So she needs at least 9 types of flower.

Answer: 9

Tip: Choosing flower A then flower B gives the same posy as B then A , so halve $n(n-1)$.

Marks: 1 mark for a correct method to count pairs, e.g. $\frac{8 \times 7}{2}$ or $\frac{9 \times 8}{2}$. 1 mark for 9.

5 (4 marks)

A bag contains 3 red, 4 blue and 5 green counters.

Two counters are taken at random without replacement.

Work out the probability that the counters are different colours.

There are 12 counters. No replacement, so the second pick is out of 11.

It is quicker to find $P(\text{same colour})$ first.

$$\text{Both red: } \frac{3}{12} \times \frac{2}{11} = \frac{6}{132}$$

$$\text{Both blue: } \frac{4}{12} \times \frac{3}{11} = \frac{12}{132}$$

$$\text{Both green: } \frac{5}{12} \times \frac{4}{11} = \frac{20}{132}$$

$$P(\text{same}) = \frac{6+12+20}{132} = \frac{38}{132}$$

$$P(\text{different}) = 1 - \frac{38}{132} = \frac{94}{132}$$

Answer: $\frac{94}{132} = \frac{47}{66}$

Tip: This is without replacement, so the second fraction is out of 11, not 12. If you work out “different” directly, there are 6 orders (RB, BR, RG, GR, BG, GB).

Marks: 1 mark for one correct product with 11 on the bottom. 1 mark for all three same-colour products (or all six different-colour products). 1 mark for $1 - \frac{38}{132}$ or a full sum. 1 mark for $\frac{94}{132}$ or an equivalent.

6 (3 marks)

9 people volunteer to run a stall at a charity event.

5 of the volunteers have a first-aid certificate.

2 of the volunteers are chosen at random.

Work out the probability that at least one of the two volunteers chosen has a first-aid certificate.

“At least one” is everything except “neither”.

4 of the 9 volunteers do not have a certificate.

Neither has one: $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

$$P(\text{at least one}) = 1 - \frac{12}{72} = \frac{60}{72}$$

Answer: $\frac{60}{72} \left(= \frac{5}{6} \right)$

Tip: The same person cannot be chosen twice, so the second fraction is out of 8. Using $1 - P(\text{neither})$ is quicker than adding three cases.

Marks: 1 mark for $\frac{4}{9} \times \frac{3}{8}$. 1 mark for $1 - \frac{12}{72}$. 1 mark for $\frac{60}{72}$ or an equivalent.

7 (4 marks)

A bag contains 4 yellow beads and 6 purple beads.

Tobias is going to take two beads at random from the bag.

Tobias says,

“If I put the first bead back before I take the second bead, I am more likely to get two purple beads than if I do not put it back.”

Is Tobias correct?

You must show how you get your answer.

Putting the first bead back: the second pick is still out of 10

$$P(\text{two purple}) = \frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = 0.36$$

Not putting it back: after a purple bead there are 5 purple beads out of 9

$$P(\text{two purple}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = 0.333 \dots$$

$0.36 > 0.333 \dots$, so two purple beads is more likely when the first bead is put back.

Tobias is correct.

Answer: Yes: $\frac{36}{100} = 0.36 > \frac{30}{90} = 0.33 \dots$

Tip: Work out both probabilities and compare them in the same form, then say clearly whether Tobias is correct.

Marks: 1 mark for $\frac{6}{10} \times \frac{6}{10}$. 1 mark for $\frac{6}{10} \times \frac{5}{9}$. 1 mark for 0.36 and 0.33 ... (or both fractions in a comparable form). 1 mark for a correct conclusion from correct probabilities.

8 (2 marks)

Two fair dice are rolled.

Given that the total is 8, find the probability that at least one dice shows a 6

We are told the total is 8, so only look at the outcomes with total 8:

(2,6), (3,5), (4,4), (5,3), (6,2)

That is 5 outcomes, all equally likely.

2 of them contain a 6: (2,6) and (6,2).

Answer: $\frac{2}{5}$

Marks: 1 mark for the 5 outcomes with total 8 (or a fraction out of 5). 1 mark for $\frac{2}{5}$.
