

Probability Equations

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

A bag contains only red, blue and green counters.

number of red counters : number of blue counters : number of green counters = 2:3: x

A counter is taken at random from the bag.

The probability that the counter is blue is $\frac{1}{4}$

Find the value of x .

Total number of parts in the ratio = $2 + 3 + x = 5 + x$

3 of these parts are blue.

$$\text{So } P(\text{blue}) = \frac{3}{5+x}$$

$$\frac{3}{5+x} = \frac{1}{4}$$

Cross-multiply: $12 = 5 + x$

$$x = 7$$

$$\text{Check: } \frac{3}{2+3+7} = \frac{3}{12} = \frac{1}{4} \quad \checkmark$$

Answer: 7

Tip: $5 + x$ is not the same as $5x$. Keep the total as $5 + x$ (in a bracket if you multiply it).

Marks: 1 mark for the total $5 + x$. 1 mark for $\frac{3}{5+x} = \frac{1}{4}$. 1 mark for 7.

2 (5 marks)

There are n pens in a box. 3 of the pens are red. The rest of the pens are black.
Kim takes at random a pen from the box. She does not put the pen back.
Kim then takes at random another pen from the box.
The probability that Kim takes a black pen and then a red pen is $\frac{1}{4}$

- (a) Show that $n^2 - 13n + 36 = 0$

There are 3 red pens and $n - 3$ black pens.

First pen black: $\frac{n-3}{n}$

Kim does not replace it, so $n - 1$ pens are left and all 3 red pens are still there.

Second pen red: $\frac{3}{n-1}$

$$\frac{n-3}{n} \times \frac{3}{n-1} = \frac{1}{4}$$

$$\frac{3(n-3)}{n(n-1)} = \frac{1}{4}$$

Cross-multiply: $12(n-3) = n(n-1)$

$$12n - 36 = n^2 - n$$

$$\text{So } n^2 - 13n + 36 = 0 \quad \checkmark$$

Answer: Shown

Tip: Taking a black pen first does not change the number of red pens: the second fraction is $\frac{3}{n-1}$, not $\frac{2}{n-1}$.

Marks: 1 mark for $\frac{n-3}{n} \times \frac{3}{n-1}$. 1 mark for setting it equal to $\frac{1}{4}$ and clearing the fractions. 1 mark for fully correct working to reach $n^2 - 13n + 36 = 0$.

- (b) There are more black pens than red pens in the box.
Work out the number of black pens in the box.

Factorise: $(n-4)(n-9) = 0$

$$n = 4 \text{ or } n = 9$$

$n = 4$ would give only 1 black pen, which is fewer than 3 red pens.

So $n = 9$.

$$\text{Black pens} = 9 - 3 = 6$$

Answer: 6

Tip: n is the total number of pens. Take away the 3 red pens to answer the question.

Marks: 1 mark for $n = 9$ (or both 4 and 9). 1 mark for 6.

3 (5 marks)

There are 10 counters in a bag. There are only red counters and blue counters in the bag.

x of the counters are red.

Two counters are taken at random from the bag without replacement.

The probability that the counters are different colours is $\frac{8}{15}$

Find the possible values of x .

You must show all your working.

There are x red and $10 - x$ blue counters.

$$P(\text{red then blue}) = \frac{x}{10} \times \frac{10 - x}{9}$$

$$P(\text{blue then red}) = \frac{10 - x}{10} \times \frac{x}{9}, \text{ which is the same.}$$

$$\text{So } P(\text{different colours}) = 2 \times \frac{x(10 - x)}{90} = \frac{x(10 - x)}{45}$$

$$\frac{x(10 - x)}{45} = \frac{8}{15}$$

$$\text{Multiply both sides by 45: } x(10 - x) = 24$$

$$10x - x^2 = 24$$

$$x^2 - 10x + 24 = 0$$

$$\text{Factorise: } (x - 4)(x - 6) = 0$$

$x = 4$ or $x = 6$ (both are possible)

Answer: $x = 4$ or $x = 6$

Tip: "Different colours" can happen two ways: red then blue, **or** blue then red. Add both.

Marks: 1 mark for $\frac{x}{10} \times \frac{10-x}{9}$. 1 mark for doubling it and setting it equal to $\frac{8}{15}$. 1 mark for $x^2 - 10x + 24 = 0$. 1 mark for solving. 1 mark for both 4 and 6.

4 (5 marks)

There are only lemon sweets and lime sweets in a jar.

The probability that a sweet taken at random from the jar is lime is $\frac{2}{5}$

Jo eats 5 of the lime sweets.

She then puts 3 more lemon sweets in the jar.

The probability that a sweet taken at random from the jar is now lime is $\frac{1}{4}$

How many sweets were in the jar at the start?

You must show all your working.

Let there be t sweets in the jar at the start.

Lime sweets at the start = $\frac{2}{5} \times t = \frac{2t}{5}$

After Jo eats 5 lime sweets: $\frac{2t}{5} - 5$ lime sweets.

Total after the changes: $t - 5 + 3 = t - 2$ sweets.

$$\frac{\frac{2t}{5} - 5}{t - 2} = \frac{1}{4}$$

Cross-multiply: $4 \left(\frac{2t}{5} - 5 \right) = t - 2$

$$\frac{8t}{5} - 20 = t - 2$$

Multiply by 5: $8t - 100 = 5t - 10$

$3t = 90$, so $t = 30$

Check: at the start 12 lime out of 30, and $\frac{12}{30} = \frac{2}{5}$. Afterwards 7 lime out of 28, and $\frac{7}{28} = \frac{1}{4}$ ✓

Answer: 30 sweets (12 lime and 18 lemon)

Tip: The new total is $t - 2$: 5 sweets are eaten and 3 are added. Only the number of lime sweets goes on the top of the new fraction.

Marks: 1 mark for $\frac{2t}{5}$ lime sweets (or $3l = 2m$ using l lime and m lemon). 1 mark for the new total $t - 2$ (or $l + m - 2$). 1 mark for a correct equation using $\frac{1}{4}$. 1 mark for a correct method to solve it. 1 mark for 30.

5 (8 marks)

A plate has n chocolate cupcakes and 12 vanilla cupcakes.
There are no other cupcakes on the plate.
A cupcake is taken at random from the plate.

- (a) Lena says that the probability that the cupcake is chocolate is $\frac{3}{8}$
Show that Lena is wrong.

There are n chocolate cupcakes out of $n + 12$ altogether.

If $P(\text{chocolate}) = \frac{3}{8}$, then $\frac{n}{n + 12} = \frac{3}{8}$

Cross-multiply: $8n = 3n + 36$

$5n = 36$

$n = 7.2$

The number of chocolate cupcakes must be a whole number.

So the probability cannot be $\frac{3}{8}$, and Lena is wrong.

Answer: n would be 7.2, which is not a whole number

Marks: 1 mark for $\frac{n}{n + 12} = \frac{3}{8}$. 1 mark for rearranging to $5n = 36$. 1 mark for $n = 7.2$ with a conclusion that this is not possible.

- (b) Two of the cupcakes are taken at random.

The probability that one cupcake is chocolate and the other is vanilla is $\frac{2}{5}$

There are fewer chocolate cupcakes than vanilla cupcakes.

Work out the number of chocolate cupcakes on the plate.

You must show all your working.

$$\text{Chocolate then vanilla: } \frac{n}{n+12} \times \frac{12}{n+11}$$

$$\text{Vanilla then chocolate: } \frac{12}{n+12} \times \frac{n}{n+11}$$

$$\text{One of each: } \frac{24n}{(n+12)(n+11)} = \frac{2}{5}$$

$$\text{Cross-multiply: } 120n = 2(n+12)(n+11), \text{ so } 60n = (n+12)(n+11)$$

$$\text{Expand: } 60n = n^2 + 23n + 132$$

$$\text{Collect terms: } n^2 - 37n + 132 = 0$$

$$\text{Factorise: } (n-4)(n-33) = 0$$

$$n = 4 \text{ or } n = 33$$

There are fewer chocolate cupcakes than vanilla (12), so $n = 4$.

$$\text{Check: } 2 \times \frac{4}{16} \times \frac{12}{15} = \frac{96}{240} = \frac{2}{5} \quad \checkmark$$

Answer: 4

Tip: "One of each" can happen in two orders, so double the product. Then use the extra fact to choose between 4 and 33.

Marks: 1 mark for $\frac{n}{n+12} \times \frac{12}{n+11}$. 1 mark for doubling it and setting it equal to $\frac{2}{5}$. 1 mark for $n^2 - 37n + 132 = 0$. 1 mark for solving. 1 mark for 4 only.

6 (5 marks)

A tin contains x red buttons, $2x$ blue buttons and 3 green buttons.

Nadia takes at random two buttons from the tin.

Find, in terms of x , an expression for the probability that Nadia takes two buttons of different colours.

Give your answer as a single fraction in its simplest form.

$$\text{Total number of buttons} = x + 2x + 3 = 3x + 3$$

It is easier to find the probability of the **same** colour first.

$$\text{Both red: } \frac{x}{3x+3} \times \frac{x-1}{3x+2}$$

$$\text{Both blue: } \frac{2x}{3x+3} \times \frac{2x-1}{3x+2}$$

$$\text{Both green: } \frac{3}{3x+3} \times \frac{2}{3x+2}$$

$$\text{Same colour: } \frac{x^2 - x + 4x^2 - 2x + 6}{(3x+3)(3x+2)} = \frac{5x^2 - 3x + 6}{9x^2 + 15x + 6}$$

$$\begin{aligned} \text{Different colours} &= 1 - \frac{5x^2 - 3x + 6}{9x^2 + 15x + 6} = \frac{9x^2 + 15x + 6 - 5x^2 + 3x - 6}{9x^2 + 15x + 6} \\ &= \frac{4x^2 + 18x}{9x^2 + 15x + 6} \end{aligned}$$

$$\text{Factorise top and bottom: } \frac{2x(2x+9)}{3(3x^2+5x+2)} = \frac{2x(2x+9)}{3(x+1)(3x+2)}$$

$$\textbf{Answer: } \frac{4x^2 + 18x}{9x^2 + 15x + 6} \left(= \frac{2x(2x+9)}{3(x+1)(3x+2)} \right)$$

Tip: “Different colours” has six orders (red-blue, blue-red, and so on). Using $1 - P(\text{same colour})$ needs only three products.

Marks: 1 mark for the total $3x + 3$. 1 mark for one correct product. 1 mark for all three same-colour products added (or all six different-colour products). 1 mark for $1 -$ your same-colour fraction. 1 mark for the final answer.
