

Parallel and Perpendicular Lines

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

The line L_1 passes through the points $(-1, 2)$ and $(3, 10)$

The line L_2 has equation $4x - 2y = 7$

Show that L_1 and L_2 are parallel.

$$\text{Gradient of } L_1 = \frac{10 - 2}{3 - (-1)} = \frac{8}{4} = 2$$

$$\text{Rearrange } L_2: -2y = -4x + 7$$

$$\text{Divide by } -2: y = 2x - \frac{7}{2}, \text{ so } L_2 \text{ has gradient } 2.$$

Both lines have gradient 2, so they are parallel.

Answer: Both lines have gradient 2, so they are parallel.

Tip: When you divide $-2y = -4x + 7$ by -2 , every sign changes. The gradient of L_2 is 2, not -2 or 4.

Marks: 1 mark for one correct gradient. 1 mark for showing both gradients are 2, with a conclusion.

2 (2 marks)

The line L passes through $(3, 1)$ and is parallel to the line $y = 5 - 2x$

Find an equation for L .

$$y = 5 - 2x \text{ has gradient } -2 \text{ (the number in front of } x\text{).}$$

$$\text{Parallel, so } L \text{ is } y = -2x + c$$

$$\text{Substitute } (3, 1): 1 = -2 \times 3 + c$$

$$1 = -6 + c, \text{ so } c = 7$$

Answer: $y = -2x + 7$

Marks: 1 mark for $y = -2x + c$ with $(3, 1)$ substituted. 1 mark for $y = -2x + 7$.

3 (2 marks)

Here are the equations of three lines.

$$A: y = 3x + 2 \quad B: 3y = -x + 6 \quad C: 2y = x - 1$$

Which two lines are perpendicular?

Give a reason for your answer.

Write each in the form $y = mx + c$ to find the gradients.

$$A: y = 3x + 2, \text{ gradient } 3$$

$$B: y = -\frac{1}{3}x + 2, \text{ gradient } -\frac{1}{3}$$

$$C: y = \frac{1}{2}x - \frac{1}{2}, \text{ gradient } \frac{1}{2}$$

$$3 \times \left(-\frac{1}{3}\right) = -1, \text{ so } A \text{ and } B \text{ are perpendicular.}$$

Answer: A and B , because $3 \times \left(-\frac{1}{3}\right) = -1$

Marks: 1 mark for finding the gradients. 1 mark for A and B with a reason.

4 (3 marks)

The line L_1 has equation $2x + 3y = 6$

The points $A(1, k)$ and $B(5, 9)$ lie on the line L_2 .

L_1 is perpendicular to L_2

Find the value of k .

You must show all your working.

$$\text{Rearrange } L_1: 3y = -2x + 6, \text{ so } y = -\frac{2}{3}x + 2$$

$$\text{The gradient of } L_1 \text{ is } -\frac{2}{3}.$$

So the gradient of L_2 is $\frac{3}{2}$ (negative reciprocal).

$$\text{Gradient of } AB = \frac{9-k}{5-1} = \frac{9-k}{4}$$

$$\frac{9-k}{4} = \frac{3}{2}$$

$$9-k = 6, \text{ so } k = 3$$

Answer: $k = 3$

Tip: The gradient of $2x + 3y = 6$ is not 2. Make y the subject first: $y = -\frac{2}{3}x + 2$.

Marks: 1 mark for the gradient of L_1 . 1 mark for an equation in k using $\frac{3}{2}$. 1 mark for $k = 3$.

5 (3 marks)

A is the point $(-2, 5)$ and B is the point $(4, 2)$

Find an equation of the line through B that is perpendicular to AB .

$$\text{Gradient of } AB = \frac{2-5}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2}$$

Perpendicular gradient = 2 (negative reciprocal of $-\frac{1}{2}$)

$$\text{So } y = 2x + c$$

$$\text{Substitute } B(4, 2): 2 = 2 \times 4 + c$$

$$2 = 8 + c, \text{ so } c = -6$$

Answer: $y = 2x - 6$

Marks: 1 mark for the gradient of AB . 1 mark for the perpendicular gradient with a point substituted. 1 mark for $y = 2x - 6$.

6 (3 marks)

The straight line L has equation $5x + 2y = 8$

The straight line N is perpendicular to L .

L and N cross the y -axis at the same point.

Find an equation for N .

Give your answer in the form $ax + by = c$ where a , b and c are integers.

Rearrange L : $2y = -5x + 8$, so $y = -\frac{5}{2}x + 4$

L has gradient $-\frac{5}{2}$ and crosses the y -axis at $(0, 4)$.

N is perpendicular, so N has gradient $\frac{2}{5}$.

N also crosses the y -axis at $(0, 4)$: $y = \frac{2}{5}x + 4$

Multiply by 5: $5y = 2x + 20$

Rearrange: $2x - 5y = -20$

Answer: $2x - 5y = -20$ (or equivalent, e.g. $-2x + 5y = 20$)

Tip: Clear the fraction by multiplying every term by 5 before you rearrange into the form $ax + by = c$.

Marks: 1 mark for the gradient $\frac{2}{5}$. 1 mark for $y = \frac{2}{5}x + 4$. 1 mark for a correct equation in the form $ax + by = c$.

7 (3 marks)

The point $P(-3, 6)$ lies on the circle $x^2 + y^2 = 45$

The line T is the tangent to the circle at P .

Find the coordinates of the point where T crosses the y -axis.

The circle has centre $O(0, 0)$.

A tangent is perpendicular to the radius at the point where it touches.

Gradient of radius $OP = \frac{6 - 0}{-3 - 0} = -2$

So the tangent has gradient $\frac{1}{2}$.

$$T: y = \frac{1}{2}x + c$$

$$\text{Substitute } P(-3, 6): 6 = \frac{1}{2} \times (-3) + c$$

$$6 = -1.5 + c, \text{ so } c = 7.5$$

T crosses the y -axis at $(0, c)$.

Answer: $(0, 7.5)$

Tip: Start with the gradient of the radius OP , then take the negative reciprocal. Take care with the negative x -coordinate: $\frac{6}{-3} = -2$.

Marks: 1 mark for the gradient of OP . 1 mark for the tangent gradient $\frac{1}{2}$ with P substituted. 1 mark for $(0, 7.5)$.

8 (3 marks)

The point P has coordinates $(2, 9)$

The point Q lies on the line $y = 3x$

The line PQ is perpendicular to the line $x + 2y = 10$

Find the coordinates of Q .

$$\text{Rearrange } x + 2y = 10: y = -\frac{1}{2}x + 5, \text{ so its gradient is } -\frac{1}{2}.$$

PQ is perpendicular to it, so the gradient of PQ is 2.

Q is on $y = 3x$, so Q is $(a, 3a)$ for some number a .

$$\text{Gradient of } PQ: \frac{3a - 9}{a - 2} = 2$$

$$3a - 9 = 2(a - 2) = 2a - 4$$

$$a = 5, \text{ so } Q \text{ is } (5, 15)$$

$$\text{Check: gradient of } PQ = \frac{15 - 9}{5 - 2} = \frac{6}{3} = 2 \quad \checkmark$$

Answer: $(5, 15)$

Marks: 1 mark for the gradient 2. 1 mark for a correct equation, e.g. $\frac{3a-9}{a-2} = 2$. 1 mark for $(5, 15)$.

9 (5 marks)

The straight line L has equation $y = 2x - 2$

The point $P(4, 6)$ lies on L .

The straight line M is perpendicular to L and passes through P .

M crosses the x -axis at the point Q .

Find the exact length of PQ .

You must show all your working.

L has gradient 2, so M has gradient $-\frac{1}{2}$.

$$M: y = -\frac{1}{2}x + c$$

Substitute $P(4, 6)$: $6 = -2 + c$, so $c = 8$

$$M \text{ is } y = -\frac{1}{2}x + 8$$

On the x -axis $y = 0$: $0 = -\frac{1}{2}x + 8$, so $x = 16$

Q is $(16, 0)$

Use Pythagoras: horizontal distance $16 - 4 = 12$, vertical distance $6 - 0 = 6$

$$PQ^2 = 12^2 + 6^2 = 144 + 36 = 180$$

$$PQ = \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}$$

Answer: $6\sqrt{5}$ (or $\sqrt{180}$)

Tip: "Exact" means leave the answer as a square root (a surd). Do not round it to a decimal.

Marks: 1 mark for the gradient $-\frac{1}{2}$. 1 mark for the equation of M . 1 mark for $Q(16, 0)$. 1 mark for $12^2 + 6^2$. 1 mark for $\sqrt{180}$ or $6\sqrt{5}$.