

Operations with Algebraic Fractions

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (4 marks)

Write as a single fraction in its simplest form.

(a) $\frac{2x}{3} + \frac{x-1}{4}$

The common denominator is 12.

$$\frac{2x}{3} = \frac{8x}{12}$$

$$\frac{x-1}{4} = \frac{3(x-1)}{12} = \frac{3x-3}{12}$$

Add the numerators: $8x + 3x - 3 = 11x - 3$

Answer: $\frac{11x-3}{12}$

Marks: 1 mark for using the common denominator 12. 1 mark for $\frac{11x-3}{12}$.

(b) $\frac{5}{x} - \frac{2}{3x}$

The common denominator is $3x$.

$$\frac{5}{x} = \frac{15}{3x}$$

Subtract the numerators: $\frac{15}{3x} - \frac{2}{3x} = \frac{13}{3x}$

Answer: $\frac{13}{3x}$

Marks: 1 mark for using the common denominator $3x$. 1 mark for $\frac{13}{3x}$.

2 (3 marks)

Express $\frac{4}{x+3} + \frac{2}{x-1}$ as a single fraction in its simplest form.

The common denominator is $(x+3)(x-1)$.

$$\frac{4}{x+3} = \frac{4(x-1)}{(x+3)(x-1)}$$

$$\frac{2}{x-1} = \frac{2(x+3)}{(x+3)(x-1)}$$

Add the numerators:

$$4(x-1) + 2(x+3) = 4x - 4 + 2x + 6 = 6x + 2$$

You can leave the denominator in brackets.

Answer: $\frac{6x+2}{(x+3)(x-1)}$

Tip: Multiply each numerator by the **other** fraction's denominator. Don't just keep 4 and 2 on the top.

Marks: 1 mark for the common denominator. 1 mark for $4(x-1) + 2(x+3)$. 1 mark for the correct answer.

3 (4 marks)

Express $\frac{3}{x-2} - \frac{x+10}{x^2-4}$ as a single fraction in its simplest form.

Factorise $x^2 - 4$ (difference of two squares): $(x-2)(x+2)$

So the common denominator is $(x-2)(x+2)$.

$$\frac{3}{x-2} = \frac{3(x+2)}{(x-2)(x+2)}$$

Subtract the numerators:

$$3(x+2) - (x+10) = 3x+6-x-10 = 2x-4$$

Factorise the top: $2x-4 = 2(x-2)$

$$\frac{2(x-2)}{(x-2)(x+2)}$$

Cancel the common factor $(x-2)$.

Answer: $\frac{2}{x+2}$

Tip: Put brackets round $(x+10)$ when you subtract it: $-(x+10) = -x-10$.

Marks: 1 mark for $x^2-4 = (x-2)(x+2)$. 1 mark for using the common denominator. 1 mark for $2x-4$. 1 mark for $\frac{2}{x+2}$.

4 (3 marks)

Simplify fully $\frac{2x^2+6x}{x^2-9} \times \frac{x-3}{4x}$

Factorise: $2x^2+6x = 2x(x+3)$

Factorise: $x^2-9 = (x+3)(x-3)$

$$\frac{2x(x+3)}{(x+3)(x-3)} \times \frac{x-3}{4x}$$

Cancel $(x+3)$ and $(x-3)$:

$$\frac{2x}{4x} = \frac{1}{2}$$

Answer: $\frac{1}{2}$

Marks: 1 mark for factorising. 1 mark for cancelling correctly. 1 mark for $\frac{1}{2}$.

5 (3 marks)

Show that $\frac{x^2 - 1}{x^2 + 3x} \div \frac{x+1}{x}$ simplifies to $\frac{x-1}{x+3}$

To divide by a fraction, flip it and multiply:

$$\frac{x^2 - 1}{x^2 + 3x} \times \frac{x}{x+1}$$

Factorise: $x^2 - 1 = (x+1)(x-1)$

Factorise: $x^2 + 3x = x(x+3)$

$$\frac{(x+1)(x-1)}{x(x+3)} \times \frac{x}{x+1}$$

Cancel $(x+1)$ and x :

$$\frac{x-1}{x+3}, \text{ as required.}$$

Answer: Shown

Tip: Factorise before you multiply. Multiplying everything out makes it very hard to cancel.

Marks: 1 mark for flipping the second fraction and multiplying. 1 mark for factorising. 1 mark for cancelling to reach the given answer.

6 (3 marks)

Show that $\frac{3}{x+2} + \frac{1}{2x+4}$ can be written in the form $\frac{a}{bx+c}$ where a , b and c are integers.

Factorise the second denominator: $2x+4 = 2(x+2)$

So the lowest common denominator is $2(x+2)$.

$$\frac{3}{x+2} = \frac{6}{2(x+2)}$$

$$\frac{6}{2(x+2)} + \frac{1}{2(x+2)} = \frac{7}{2(x+2)}$$

$$= \frac{7}{2x+4}$$

This is in the form $\frac{a}{bx+c}$, with $a = 7$, $b = 2$ and $c = 4$.

Answer: $\frac{7}{2x+4}$

Tip: Multiplying the denominators gives $(x+2)(2x+4)$, which works but is not the simplest. Look for a common factor first: $2x+4 = 2(x+2)$.

Marks: 1 mark for spotting $2x+4 = 2(x+2)$ or for a correct common denominator. 1 mark for $\frac{6}{2(x+2)} + \frac{1}{2(x+2)}$ (or an equivalent correct sum). 1 mark for $\frac{7}{2x+4}$.

7 (3 marks)

Express $\frac{2}{x+1} + \frac{3}{x-3} - \frac{5}{x}$ as a single fraction in its simplest form.

The common denominator is $x(x+1)(x-3)$.

Multiply each numerator by the brackets its denominator is missing:

$$2 \times x(x-3) = 2x^2 - 6x$$

$$3 \times x(x+1) = 3x^2 + 3x$$

$$5 \times (x+1)(x-3) = 5(x^2 - 2x - 3) = 5x^2 - 10x - 15$$

$$\text{Numerator: } (2x^2 - 6x) + (3x^2 + 3x) - (5x^2 - 10x - 15)$$

$$= 2x^2 - 6x + 3x^2 + 3x - 5x^2 + 10x + 15$$

$$= 7x + 15$$

Answer: $\frac{7x+15}{x(x+1)(x-3)}$

Marks: 1 mark for the common denominator. 1 mark for the numerator with at most one error. 1 mark for the correct answer.

8 (4 marks)

Given that

$$\frac{x+k}{x+3} - \frac{2}{x-1} = \frac{x^2+2x-11}{(x+3)(x-1)}$$

find the value of k .

Write the left-hand side over the common denominator $(x+3)(x-1)$:

$$\frac{(x+k)(x-1) - 2(x+3)}{(x+3)(x-1)}$$

Expand the top: $x^2 - x + kx - k - 2x - 6$

$$= x^2 + (k-3)x - (k+6)$$

Compare with $x^2 + 2x - 11$:

$$x \text{ terms: } k-3 = 2, \text{ so } k = 5$$

$$\text{Check the numbers: } -(5+6) = -11 \quad \checkmark$$

Answer: $k = 5$

Tip: Put brackets round each part when you expand. $-2(x+3) = -2x - 6$: both terms are subtracted.

Marks: 1 mark for $(x+k)(x-1) - 2(x+3)$ over $(x+3)(x-1)$. 1 mark for expanding correctly. 1 mark for comparing coefficients, e.g. $k-3 = 2$. 1 mark for $k = 5$.

9 (3 marks)

Show that $\frac{x^2-16}{5x^3} \div \frac{x+4}{10x}$ can be written in the form $\frac{a(x+b)}{x^2}$ where a and b are integers.

To divide, flip the second fraction and multiply: $\frac{x^2-16}{5x^3} \times \frac{10x}{x+4}$

$x^2 - 16$ is a difference of two squares: $(x+4)(x-4)$

$$\frac{(x+4)(x-4)}{5x^3} \times \frac{10x}{x+4}$$

$$\text{Cancel } (x+4): \frac{10x(x-4)}{5x^3}$$

Cancel $5x$: $\frac{2(x-4)}{x^2}$

This is in the form $\frac{a(x+b)}{x^2}$, with $a = 2$ and $b = -4$.

Answer: $\frac{2(x-4)}{x^2}$

Tip: Flip the second fraction **before** you cancel anything. Then cancel the x in $10x$ with one of the x s in x^3 .

Marks: 1 mark for $(x+4)(x-4)$. 1 mark for turning the division into a multiplication and cancelling $(x+4)$. 1 mark for $\frac{2(x-4)}{x^2}$.
