

nth Term of Quadratic Sequences

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (3 marks)

Here are the first five terms of a quadratic sequence.

4 10 18 28 40

(a) Find the next term of the sequence.

First differences: 6, 8, 10, 12

These go up by 2, so the next difference is 14.

Next term = $40 + 14 = 54$

Answer: 54

(b) Find an expression, in terms of n , for the n th term of the sequence.

Second difference: 2

Half of 2 is 1, so the sequence starts with n^2 .

n^2 gives 1, 4, 9, 16, 25

Subtract these from the sequence:

$4 - 1 = 3$, $10 - 4 = 6$, $18 - 9 = 9$, $28 - 16 = 12$, $40 - 25 = 15$

3, 6, 9, 12, 15 is the linear sequence $3n$.

Answer: $n^2 + 3n$

Marks: 1 mark for n^2 . 1 mark for $n^2 + 3n$.

2 (3 marks)

Here are the first five terms of a quadratic sequence.

1 3 7 13 21

Find an expression, in terms of n , for the n th term of this sequence.

First differences: 2, 4, 6, 8

Second difference: 2, so the sequence starts with n^2 .

Subtract n^2 from each term:

$$1 - 1 = 0, \quad 3 - 4 = -1, \quad 7 - 9 = -2, \quad 13 - 16 = -3, \quad 21 - 25 = -4$$

0, -1, -2, -3, -4 goes down by 1 each time, so it is $-n + 1$.

So the n th term is $n^2 - n + 1$

Check with $n = 3$: $9 - 3 + 1 = 7$ ✓

Answer: $n^2 - n + 1$

Tip: Check your answer by substituting $n = 1, 2$ and 3 . This catches sign mistakes, such as writing $n^2 + n + 1$.

Marks: 1 mark for n^2 . 1 mark for finding the linear part $-n + 1$. 1 mark for the correct answer.

3 (3 marks)

Find the n th term of the sequence

7 17 33 55 83

First differences: 10, 16, 22, 28

Second difference: 6

Half of 6 is 3, so the sequence starts with $3n^2$.

$3n^2$ gives 3, 12, 27, 48, 75

Subtract these from the sequence:

$$7 - 3 = 4, \quad 17 - 12 = 5, \quad 33 - 27 = 6, \quad 55 - 48 = 7, \quad 83 - 75 = 8$$

4, 5, 6, 7, 8 is the linear sequence $n + 3$.

Answer: $3n^2 + n + 3$

Tip: The number in front of n^2 is **half** the second difference: a second difference of 6 gives $3n^2$, not $6n^2$.

Marks: 1 mark for $3n^2$. 1 mark for finding the linear part $n + 3$. 1 mark for the correct answer.

4 (3 marks)

The n th term of a quadratic sequence is $an^2 + bn$

The first term is 5 and the second term is 14

Find the values of a and b .

First term ($n = 1$): $a + b = 5$

Second term ($n = 2$): $4a + 2b = 14$

Divide the second equation by 2: $2a + b = 7$

Subtract $a + b = 5$ from $2a + b = 7$: $a = 2$

Substitute into $a + b = 5$: $b = 3$

Check: $2 \times 4 + 3 \times 2 = 14$ ✓

Answer: $a = 2$, $b = 3$

Marks: 1 mark for one correct equation. 1 mark for a correct method to solve the equations. 1 mark for both values.

5 (2 marks)

The n th term of a sequence is $2n^2 - 3n$

Show that 104 is a term of the sequence.

Set the n th term equal to 104: $2n^2 - 3n = 104$

Rearrange: $2n^2 - 3n - 104 = 0$

Factorise: $(2n + 13)(n - 8) = 0$

So $n = 8$ or $n = -6.5$

$n = 8$ is a positive whole number, so 104 is the 8th term.

Check: $2 \times 64 - 3 \times 8 = 128 - 24 = 104$ ✓

Answer: 104 is the 8th term.

Marks: 1 mark for forming and solving $2n^2 - 3n = 104$. 1 mark for $n = 8$ with a conclusion.

6 (3 marks)

Here are the first six terms of a quadratic sequence.

2 9 20 35 54 77

Find an expression, in terms of n , for the n th term of this sequence.

First differences: 7, 11, 15, 19, 23

Second difference: 4

Half of 4 is 2, so the sequence starts with $2n^2$.

$2n^2$ gives 2, 8, 18, 32, 50, 72

Subtract these from the sequence:

0, 1, 2, 3, 4, 5

This goes up by 1 and starts at 0, so it is $n - 1$.

Answer: $2n^2 + n - 1$

Tip: Many students find the $2n^2$ and then get stuck. Always subtract it from each term and find the n th term of what is left.

Marks: 1 mark for $2n^2$. 1 mark for finding the linear part $n - 1$. 1 mark for the correct answer.

7 (4 marks)

Kofi makes a sequence of patterns using square tiles.

The number of tiles in Pattern n is $pn^2 + qn$, where p and q are constants.

Pattern 2 uses 14 tiles.

Pattern 5 uses 65 tiles.

Kofi has 200 tiles.

What is the number of the largest pattern he can make?

Pattern 2 ($n = 2$): $4p + 2q = 14$, so $2p + q = 7$

Pattern 5 ($n = 5$): $25p + 5q = 65$, so $5p + q = 13$

Subtract the first from the second: $3p = 6$, so $p = 2$

Substitute into $2p + q = 7$: $4 + q = 7$, so $q = 3$

So Pattern n uses $2n^2 + 3n$ tiles.

Pattern 9: $2 \times 81 + 3 \times 9 = 162 + 27 = 189$ tiles (189 is less than 200)

Pattern 10: $2 \times 100 + 3 \times 10 = 200 + 30 = 230$ tiles (too many)

So the largest pattern Kofi can make is Pattern 9.

Answer: Pattern 9

Tip: Check the patterns either side of 200 tiles: Pattern 9 needs 189 tiles and Pattern 10 needs 230, so Pattern 10 is not possible.

Marks: 1 mark for one correct equation in p and q . 1 mark for finding p or q . 1 mark for $2n^2 + 3n$. 1 mark for Pattern 9 with 189 (or 230) shown.

8 (3 marks)

The n th term of sequence A is $n^2 + 4n$

The n th term of sequence B is $12n - 7$

For two values of n , the n th term of sequence A is equal to the n th term of sequence B .

Find these two values of n .

You must show all your working.

Set the n th terms equal: $n^2 + 4n = 12n - 7$

Rearrange: $n^2 - 8n + 7 = 0$

Factorise: $(n - 1)(n - 7) = 0$

So $n = 1$ or $n = 7$

Check $n = 7$: $A: 49 + 28 = 77$, $B: 84 - 7 = 77$ ✓

Answer: $n = 1$ and $n = 7$

Tip: Collect all the terms on one side so the quadratic equals 0 before you factorise.

Marks: 1 mark for $n^2 + 4n = 12n - 7$. 1 mark for $n^2 - 8n + 7 = 0$ factorised or solved correctly. 1 mark for $n = 1$ and $n = 7$.
