

Negative and Fractional Indices

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (5 marks)

- (a) Write down the value of 7^0

Any number (except 0) to the power 0 is 1.

Answer: 1

Tip: 7^0 is not 0 and it is not 7. Anything to the power 0 is 1.

- (b) Find the value of $81^{-\frac{1}{2}}$

The minus sign means “one over”. The $\frac{1}{2}$ means square root.

$$81^{-\frac{1}{2}} = \frac{1}{\sqrt{81}} = \frac{1}{9}$$

Answer: $\frac{1}{9}$

- (c) Find the value of $125^{\frac{2}{3}}$

The bottom of the fraction (3) means cube root. The top (2) means square.

$$\sqrt[3]{125} = 5$$

$$5^2 = 25$$

Answer: 25

Tip: After the cube root, **square** the answer: $5^2 = 25$. Do not multiply by 2.

- (d) Find the value of $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

The minus sign means flip the fraction:

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}}$$

Fourth root: $\sqrt[4]{\frac{81}{16}} = \frac{3}{2}$ (because $3^4 = 81$ and $2^4 = 16$)

Cube: $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

Answer: $\frac{27}{8}$

Marks: 1 mark for $\frac{81}{16}$ or $\frac{3}{2}$ or $\frac{8}{27}$. 1 mark for $\frac{27}{8}$.

2 (2 marks)

$$2^n = \frac{\sqrt{2}}{8}$$

Find the value of n .

Write each part as a power of 2:

$$\sqrt{2} = 2^{\frac{1}{2}} \text{ and } 8 = 2^3$$

Dividing, so subtract the powers:

$$\frac{2^{\frac{1}{2}}}{2^3} = 2^{\frac{1}{2}-3} = 2^{-\frac{5}{2}}$$

So $n = -\frac{5}{2}$

Answer: $-\frac{5}{2}$

Marks: 1 mark for $2^{\frac{1}{2}}$ and 2^3 (or 2^{-3}). 1 mark for $-\frac{5}{2}$.

3 (2 marks)

Write $25^n \times 125^{1-n}$ as a power of 5

Write 25 and 125 as powers of 5: $25 = 5^2$ and $125 = 5^3$

$$25^n = (5^2)^n = 5^{2n}$$

$$125^{1-n} = (5^3)^{1-n} = 5^{3(1-n)} = 5^{3-3n}$$

Multiplying, so add the powers:

$$2n + 3 - 3n = 3 - n$$

So the answer is 5^{3-n}

Answer: 5^{3-n}

Tip: Multiply both terms in the bracket by 3: $3(1-n) = 3-3n$, not $3-n$.

Marks: 1 mark for 5^{2n} and 5^{3-3n} (or $5^{3(1-n)}$). 1 mark for 5^{3-n} .

4 (2 marks)

Simplify fully $\left(\frac{27x^6}{y^3}\right)^{-\frac{1}{3}}$

Deal with the $\frac{1}{3}$ first (cube root of each part):

$$\sqrt[3]{27} = 3, (x^6)^{\frac{1}{3}} = x^2, (y^3)^{\frac{1}{3}} = y$$

$$\text{So } \left(\frac{27x^6}{y^3}\right)^{\frac{1}{3}} = \frac{3x^2}{y}$$

The minus sign means flip the fraction:

$$\left(\frac{3x^2}{y}\right)^{-1} = \frac{y}{3x^2}$$

Answer: $\frac{y}{3x^2}$

Tip: Apply the power to every part: the 27, the x^6 and the y^3 .

Marks: 1 mark for $\frac{3x^2}{y}$ or the flipped fraction $\frac{y^3}{27x^6}$. 1 mark for $\frac{y}{3x^2}$.

5 (2 marks)

$$\frac{9^a}{3^b} = 27$$

Show that $b = 2a - 3$

Write everything as a power of 3:

$$9^a = (3^2)^a = 3^{2a} \quad \text{and} \quad 27 = 3^3$$

$$\text{So } \frac{3^{2a}}{3^b} = 3^3$$

Dividing, so subtract the powers: $3^{2a-b} = 3^3$

The powers must be equal: $2a - b = 3$

Rearrange: $b = 2a - 3$ ✓

Answer: $2a - b = 3$

Marks: 1 mark for 3^{2a} and 3^3 . 1 mark for $2a - b = 3$ leading to $b = 2a - 3$.

6 (3 marks)

Work out the value of $\left(\frac{9}{4}\right)^{-\frac{1}{2}} \times \left(\frac{2}{3}\right)^{-3}$
 You must show all your working.

First part: flip the fraction, then square root.

$$\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{2}{3}$$

Second part: flip the fraction, then cube.

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

Multiply: $\frac{2}{3} \times \frac{27}{8} = \frac{54}{24}$

Simplify (divide top and bottom by 6): $\frac{9}{4}$

Answer: $\frac{9}{4}$

Tip: A negative power flips the fraction: $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$. The answer is not negative.

Marks: 1 mark for $\frac{2}{3}$. 1 mark for $\frac{27}{8}$. 1 mark for $\frac{9}{4}$ (or equivalent).

7 (3 marks)

$$2^a = \frac{1}{8} \quad 2^b = 4^5 \quad 2^c = \frac{1}{\sqrt{2}}$$

Work out the value of $a + b + c$

$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}, \text{ so } a = -3$$

$$4^5 = (2^2)^5 = 2^{10}, \text{ so } b = 10$$

$$\frac{1}{\sqrt{2}} = \frac{1}{2^{\frac{1}{2}}} = 2^{-\frac{1}{2}}, \text{ so } c = -\frac{1}{2}$$

$$a + b + c = -3 + 10 - \frac{1}{2} = 6\frac{1}{2}$$

Answer: $6\frac{1}{2}$

Marks: 1 mark for two of $a = -3$, $b = 10$, $c = -\frac{1}{2}$. 1 mark for all three. 1 mark for $6\frac{1}{2}$ (or equivalent).

8 (4 marks)

$$5^x \times 25^y = 5^7 \text{ and } 2^x \div 2^y = 2$$

Work out the values of x and y .

$$\text{First equation: } 25^y = (5^2)^y = 5^{2y}$$

$$\text{So } 5^x \times 5^{2y} = 5^{x+2y} = 5^7$$

$$\text{This gives } x + 2y = 7$$

$$\text{Second equation: } 2^x \div 2^y = 2^{x-y} = 2^1$$

$$\text{This gives } x - y = 1$$

Subtract the second equation from the first: $3y = 6$, so $y = 2$

Substitute into $x - y = 1$: $x - 2 = 1$, so $x = 3$

Answer: $x = 3$, $y = 2$

Marks: 1 mark for $x + 2y = 7$. 1 mark for $x - y = 1$. 1 mark for a correct method to solve them. 1 mark for $x = 3$ and $y = 2$.
