

Mixed Sine and Cosine Rule Problems

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (5 marks)

Here is triangle PQR .

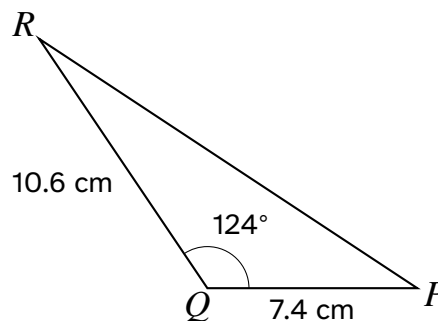


Diagram **NOT** accurately drawn

- (a) Work out the area of triangle PQR .
Give your answer correct to 3 significant figures.

The 124° angle is between the 7.4 cm and 10.6 cm sides.

$$\text{Area} = \frac{1}{2}ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 7.4 \times 10.6 \times \sin 124^\circ = 32.51 \dots$$

Answer: 32.5 cm²

Tip: $\sin 124^\circ$ is positive, so the formula works the same for an obtuse angle.

Marks: 1 mark for $\frac{1}{2} \times 7.4 \times 10.6 \times \sin 124^\circ$. 1 mark for 32.5.

- (b) Work out the length of PR .
Give your answer correct to 3 significant figures.

We know two sides and the angle between them, so use the cosine rule.

$$PR^2 = 7.4^2 + 10.6^2 - 2 \times 7.4 \times 10.6 \times \cos 124^\circ$$

$\cos 124^\circ$ is negative, so we add: $PR^2 = 167.12 + 87.72 \dots = 254.84 \dots$

$$PR = \sqrt{254.84 \dots} = 15.96 \dots$$

Answer: 16.0 cm

Tip: PR is opposite the obtuse angle, so it must be the longest side. An answer shorter than 10.6 cm means the sign of $\cos 124^\circ$ has gone wrong.

Marks: 1 mark for using the cosine rule. 1 mark for $PR^2 = 254.8 \dots$ 1 mark for 16.0.

2 (5 marks)

DEF is a triangle. $DE = 7$ cm, $DF = 11$ cm and angle $DEF = 105^\circ$

Work out the length of EF .

Give your answer correct to 3 significant figures.

The 105° angle is not between the two known sides, so we cannot use the cosine rule for EF straight away.

Step 1: sine rule for angle F . $DE = 7$ is opposite angle F and $DF = 11$ is opposite angle E .

$$\frac{\sin F}{7} = \frac{\sin 105^\circ}{11}, \text{ so } \sin F = \frac{7 \sin 105^\circ}{11} = 0.6146 \dots$$

$$\text{Angle } F = 37.92 \dots^\circ$$

Step 2: angles in a triangle: angle $D = 180 - 105 - 37.92 \dots = 37.07 \dots^\circ$

Step 3: sine rule for EF , which is opposite angle D .

$$EF = \frac{11 \sin 37.07 \dots^\circ}{\sin 105^\circ} = 6.864 \dots$$

Answer: 6.86 cm

Tip: Match each side with the angle opposite it. $DF = 11$ goes with the 105° at E , not with angle D .

Marks: 1 mark for the sine rule for angle F . 1 mark for 37.9° . 1 mark for angle $D = 37.1^\circ$. 1 mark for a correct method for EF . 1 mark for an answer from 6.86 to 6.87.

3 (5 marks)

A field ABC is a triangle where $AB = 85$ m, $AC = 120$ m and angle $BAC = 72^\circ$
Fencing costs £8.50 per metre.

Work out the cost of fencing the whole perimeter of the field.

To find the perimeter we need the third side, BC .

Angle BAC is between AB and AC , so use the cosine rule.

$$BC^2 = 85^2 + 120^2 - 2 \times 85 \times 120 \times \cos 72^\circ = 15\,321.0 \dots$$

$$BC = \sqrt{15\,321.0 \dots} = 123.77 \dots \text{ m}$$

$$\text{Perimeter} = 85 + 120 + 123.77 \dots = 328.77 \dots \text{ m}$$

$$\text{Cost} = 328.77 \dots \times 8.50 = 2794.61 \dots$$

Round to the nearest penny: £2794.62

Answer: £2794.62 (£2794 to £2795 is fine)

Marks: 1 mark for using the cosine rule. 1 mark for $BC = 123.8$. 1 mark for the perimeter. 1 mark for multiplying by 8.50. 1 mark for an answer from 2794 to 2795.

4 (5 marks)

The diagram shows the positions of two coastguard stations and a ship.

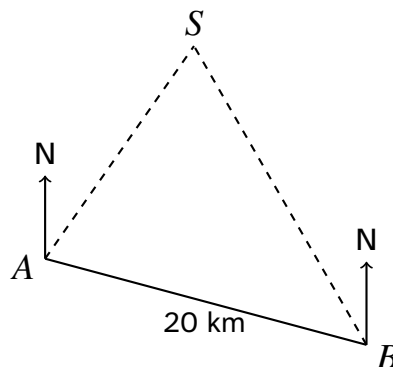


Diagram **NOT** accurately drawn

A and B are two coastguard stations. B is 20 km from A on a bearing of 105° .
A ship, S , is on a bearing of 035° from A and on a bearing of 330° from B .
A lifeboat travels in a straight line from B to the ship at an average speed of 24 km/h.
Work out how long the lifeboat takes to reach the ship.
Give your answer to the nearest minute.
You must show all your working.

$$\text{Angle } SAB = 105 - 35 = 70^\circ$$

The bearing of A from B is $105 + 180 = 285^\circ$, so angle $ABS = 330 - 285 = 45^\circ$

$$\text{Angle } ASB = 180 - 70 - 45 = 65^\circ$$

Sine rule: BS is opposite angle A and $AB = 20$ is opposite angle S .

$$BS = \frac{20 \sin 70^\circ}{\sin 65^\circ} = 20.73 \dots \text{ km}$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{20.73 \dots}{24} = 0.8640 \dots \text{ hours}$$

$$0.8640 \dots \times 60 = 51.84 \dots \text{ minutes}$$

Answer: 52 minutes

Tip: Change hours to minutes by multiplying by 60. 0.86 hours is not 86 minutes.

Marks: 1 mark for angle $SAB = 70^\circ$ or angle $ABS = 45^\circ$. 1 mark for all three angles. 1 mark for the sine rule for BS . 1 mark for $BS = 20.7$ km. 1 mark for 52 minutes.

5 (5 marks)

The diagram shows triangle PQR .

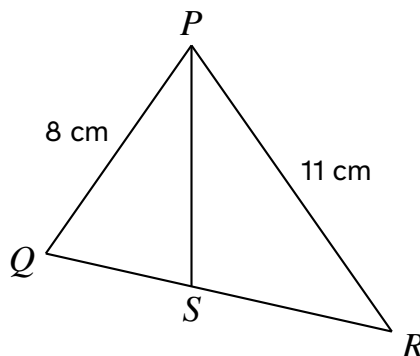


Diagram **NOT** accurately drawn

$PQ = 8$ cm, $PR = 11$ cm and angle $QPR = 70^\circ$

S is the point on QR such that angle $QPS =$ angle SPR .

Work out the length of PS .

Give your answer correct to 3 significant figures.

You must show all your working.

Step 1: cosine rule for QR (two sides and the angle between them).

$$QR^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 70^\circ = 124.80 \dots$$

$$QR = 11.17 \dots$$

Step 2: sine rule for angle PQR . $PR = 11$ is opposite angle Q .

$$\sin Q = \frac{11 \sin 70^\circ}{11.17 \dots} = 0.9252 \dots, \text{ so angle } PQR = 67.70 \dots^\circ$$

Step 3: in triangle PQS , angle $QPS = 70 \div 2 = 35^\circ$

$$\text{Angle } PSQ = 180 - 35 - 67.70 \dots = 77.29 \dots^\circ$$

Step 4: sine rule in triangle PQS . PS is opposite angle Q and $PQ = 8$ is opposite angle PSQ .

$$PS = \frac{8 \sin 67.70 \dots^\circ}{\sin 77.29 \dots^\circ} = 7.587 \dots$$

Answer: 7.59 cm

Tip: S is not the midpoint of QR , and angle PSQ is not 90° . Work in triangle PQS using the 35° angle.

Marks: 1 mark for the cosine rule for QR . 1 mark for a method for angle PQR (or angle PRQ). 1 mark for angle PSQ (or angle PSR). 1 mark for the sine rule for PS . 1 mark for an answer from 7.58 to 7.59.

6 (5 marks)

The diagram shows a plot of land $ABCD$.

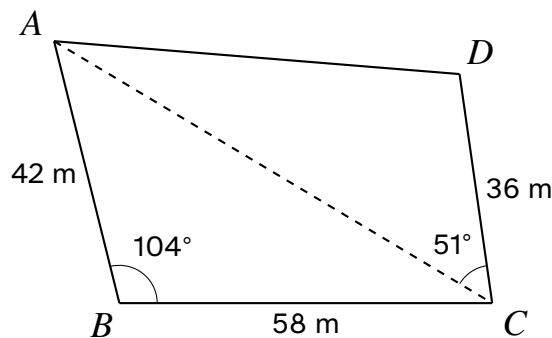


Diagram **NOT** accurately drawn

$AB = 42$ m, $BC = 58$ m and angle $ABC = 104^\circ$

$CD = 36$ m and angle $ACD = 51^\circ$

Work out the area of the plot of land.

Give your answer correct to 3 significant figures.

Split the plot along AC into triangles ABC and ACD .

Step 1: area of $ABC = \frac{1}{2} \times 42 \times 58 \times \sin 104^\circ = 1181.8 \dots \text{ m}^2$

Step 2: cosine rule for AC : $AC^2 = 42^2 + 58^2 - 2 \times 42 \times 58 \times \cos 104^\circ = 6306.6 \dots$

$AC = 79.41 \dots$ m

Step 3: angle $ACD = 51^\circ$ is between AC and CD .

Area of $ACD = \frac{1}{2} \times 79.41 \dots \times 36 \times \sin 51^\circ = 1110.8 \dots \text{ m}^2$

Total area = $1181.8 \dots + 1110.8 \dots = 2292.7 \dots$

Answer: 2290 m^2

Tip: You need AC before you can find the area of triangle ACD , because the 51° angle is between AC and CD .

Marks: 1 mark for the area of triangle ABC . 1 mark for the cosine rule for AC . 1 mark for $AC = 79.4$. 1 mark for the area of triangle ACD . 1 mark for an answer from 2290 to 2293.
