

Linear Inequalities and Set Notation

Solutions

For Pearson Edexcel GCSE Mathematics (1MA1) • Higher tier

Each question is shown in grey, followed by the working, one step per line. The final answer is in a box. *Marks* shows how the marks are usually given in the exam, and **Tip** points out a common mistake.

1 (2 marks)

Solve $7x - 4 > 3x + 10$

Subtract $3x$ from both sides:

$$4x - 4 > 10$$

Add 4 to both sides:

$$4x > 14$$

Divide both sides by 4:

$$x > 3.5$$

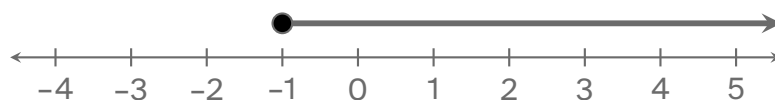
Answer: $x > 3.5$

Tip: Keep the inequality sign all the way through. Writing $x = 3.5$ as your answer loses a mark.

Marks: 1 mark for $4x > 14$. 1 mark for $x > 3.5$.

2 (2 marks)

(a) Write down the inequality shown on this number line.



The circle at -1 is filled, so -1 is included.

The arrow points to the right, so x can be any number bigger than -1 .

Use $\geq$ (greater than or equal to).

Answer: $x \geq -1$

(b) $t > -3$ and $t \leq 2$

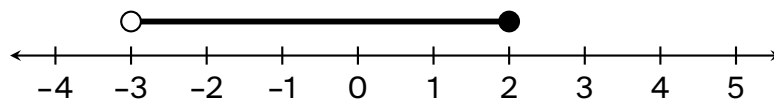
On the number line below, show all the values of t that satisfy both inequalities.

Together the two inequalities say $-3 < t \leq 2$.

-3 is not included ($>$), so draw an open circle at -3 .

2 is included ($\leq$), so draw a filled circle at 2 .

Join the two circles with a line.



Answer: Open circle at -3 , filled circle at 2 , joined

3 (5 marks)

(a) Solve $-7 < 3x - 1 \leq 8$

Do the same thing to all three parts.

Add 1 to all three parts:

$$-6 < 3x \leq 9$$

Divide all three parts by 3:

$$-2 < x \leq 3$$

Answer: $-2 < x \leq 3$

Tip: A common mistake is to add 1 to only one side. Whatever you do, do it to all three parts.

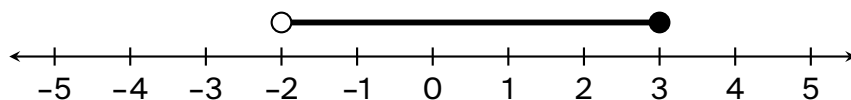
Marks: 1 mark for $-6 < 3x \leq 9$ (or for -2 and 3 seen). 1 mark for $-2 < x \leq 3$.

(b) Represent your solution to part (a) on the number line.

-2 is not included ($<$), so draw an open circle at -2 .

3 is included ($\leq$), so draw a filled circle at 3 .

Join the two circles with a line.



Answer: Open circle at -2 , filled circle at 3 , joined

Marks: 2 marks for both ends correct and joined. 1 mark if one end is correct.

(c) Write down all the integers that satisfy $-7 < 3x - 1 \leq 8$

From part (a), $-2 < x \leq 3$.

-2 is not included, but 3 is.

Answer: $-1, 0, 1, 2, 3$

4 (3 marks)

Solve $3 - \frac{2x}{5} > 7$

Subtract 3 from both sides:

$$-\frac{2x}{5} > 4$$

Multiply both sides by 5:

$$-2x > 20$$

Divide both sides by -2 .

Dividing by a negative number reverses the sign:

$$x < -10$$

Answer: $x < -10$

Tip: Remember to reverse the sign when you divide by a negative. To avoid this, add $2x$ to both sides instead: $-20 > 2x$, so $-10 > x$.

Marks: 1 mark for $-\frac{2x}{5} > 4$. 1 mark for $-2x > 20$. 1 mark for $x < -10$.

5 (3 marks)

Solve $\frac{3 - 2x}{4} \geq -2$

Write your answer using set notation.

Multiply both sides by 4:

$$3 - 2x \geq -8$$

Subtract 3 from both sides:

$$-2x \geq -11$$

Divide both sides by -2 and reverse the sign:

$$x \leq 5.5$$

In set notation: $\{x: x \leq 5.5\}$

Answer: $\{x: x \leq 5.5\}$

Marks: 1 mark for $3 - 2x \geq -8$. 1 mark for $x \leq 5.5$. 1 mark for writing your answer in set notation.

6 (2 marks)

Kwame solves the inequality $2x - 5 \geq 1$
He shows his solution on this number line.



Kwame's number line is wrong.
Give two reasons why.

First solve the inequality: $2x \geq 6$, so $x \geq 3$.

Kwame found the correct boundary value, 3.

$\geq$ means 3 is included, so the circle at 3 should be filled, not open.

$x \geq 3$ means numbers bigger than 3, so the arrow should point to the right, not the left.

Answer: The circle at 3 should be filled (3 is included); the arrow should point to the right (to numbers greater than 3)

Tip: A filled circle means the end value is included ($\leq$ or $\geq$). An open circle means it is not ($<$ or $>$).

Marks: 1 mark for each correct reason.

7 (2 marks)

Work out the smallest integer n that satisfies the inequality $4(n + 3) > 27$

Expand the bracket:

$$4n + 12 > 27$$

Subtract 12 from both sides:

$$4n > 15$$

Divide both sides by 4:

$$n > 3.75$$

n must be greater than 3.75, so the smallest integer is 4.

Answer: 4

Tip: 3 is not the answer: 3 is less than 3.75. Check it: $4(3 + 3) = 24$, which is not greater than 27.

Marks: 1 mark for $n > 3.75$. 1 mark for 4.

8 (3 marks)

$\xi = \{\text{integers from 1 to 12}\}$

$A = \{x: 2x + 3 > 11\}$ $B = \{x: 3x \leq 27\}$

List the members of $A \cap B$.

Solve the inequality for A :

$$2x > 8$$

$$x > 4$$

So $A = \{5, 6, 7, 8, 9, 10, 11, 12\}$

Solve the inequality for B :

$$x \leq 9$$

So $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cap B$ means the members that are in both A and B .

Answer: 5, 6, 7, 8, 9

Tip: $A \cap B$ (intersection) means in **both** sets. $A \cup B$ (union) means in **either** set.

Marks: 1 mark for $x > 4$. 1 mark for $x \leq 9$. 1 mark for 5, 6, 7, 8, 9.

9 (2 marks)

a and b are positive integers.

$$a + 2b = 13 \text{ and } a > b$$

Find all the possible values of a .

$$a + 2b = 13, \text{ so } a = 13 - 2b.$$

We need $a > b$:

$$13 - 2b > b$$

Add $2b$ to both sides:

$$13 > 3b$$

$b < 4.33 \dots$, so b can be 1, 2, 3 or 4.

Find $a = 13 - 2b$ for each: 11, 9, 7, 5

Check: $b = 5$ gives $a = 3$, which is not greater than 5 ✗

Answer: 11, 9, 7, 5

Marks: 1 mark for $b < 4.33 \dots$ or $b \leq 4$. 1 mark for all four values 5, 7, 9, 11 and no others.

10 (3 marks)

A rectangle has length $(2x + 3)$ cm and width x cm.

The perimeter of the rectangle is more than 24 cm and less than 48 cm.

Find the range of possible values of x .

You must show all your working.

$$\text{Perimeter} = 2 \times \text{length} + 2 \times \text{width}$$

$$= 2(2x + 3) + 2x$$

$$= 4x + 6 + 2x$$

$$= 6x + 6$$

The perimeter is between 24 and 48:

$$24 < 6x + 6 < 48$$

Subtract 6 from all three parts:

$$18 < 6x < 42$$

Divide all three parts by 6:

$$3 < x < 7$$

Answer: $3 < x < 7$

Marks: 1 mark for the perimeter $6x + 6$. 1 mark for $24 < 6x + 6 < 48$. 1 mark for $3 < x < 7$.
